Coupled Electromagnetics, Multiphase Transport and Large Deformation Model for Microwave Drying

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Abstract

The present work involves development of a fundamentals-based coupled electromagnetics, multiphase transport and large deformation model to understand microwave drying of a hygroscopic porous material. Microwave drying is carried out in a 950 W domestic microwave oven operating at 10% power level. Electric field distribution inside the oven cavity and porous material are obtained by solving Maxwell's equations for electromagnetics. Modes of fluid transport include capillarity, binary diffusion and gas pressure-driven flow. Large deformation, included by treating the solid as hyperelastic, is implemented in a novel way using the Arbitrary-Lagrangian-Eulerian framework for mesh movement. Deformation during microwave drying was found to critically alter material structure that significantly affected microwave absorption, heat and moisture transport within the material. Sensitivity analysis revealed that moisture loss and volumetric shrinkage were unaffected with changes in intrinsic permeability and elastic modulus of the material while stress state within the material was highly sensitive to elastic modulus values.

keywords: microwave drying, large deformation, ALE, multiphase transport, fluid structure interaction

Nomenclature

Symbol	Description	Units
c	concentration	kg/m^{-3}
C_p	specific heat capacity	$\text{J}/\text{kg} \cdot \text{K}$
C_g	molar density	kmol/m^3
$D_{eff,g}$	vapor diffusivity in air	m^2/s
$D_{w,cap}$	capillary diffusivity	m^2/s
E	elastic modulus	N/m^2
$\mathbf{E}_{el}$	Green-Lagrange strain tensor	
$\mathbf{E}$	electric field intensity	V/m
$\mathbf{F}$	deformation tensor	
h_t	heat transfer coefficient	$\text{W}/\text{m}^2 \cdot \text{K}$
h_m	mass transfer coefficient	m/s
$\mathbf{H}$	magnetic field intensity	A/m
$\dot{I}$	rate of evaporation	$\text{kg}/\text{m}^3 \cdot \text{s}$
$\mathbf{I}$	Identity tensor	
J	Jacobian	
$k_{in,i}$	intrinsic permeability	m^2
$k_{r,i}$	relative permeability of component i	
K_{evap}	evaporation rate constant	$1/\text{s}$
m	overall mass fraction	
M	moisture content (dry basis)	$\text{kg water}/\text{kg dry solid}$
M_a, M_v	Molecular weight of air and vapor	
$\vec{N}$	unit normal	
p	pressure	Pa
p_c	capillary pressure of water	Pa
Q	microwave source term	W/m^3
r	radius	m
R	universal gas constant	$\text{J}/\text{kmol} \cdot \text{K}$
S_i	saturation of a fluid phase, i	
$\mathbf{S}'$	Piola-Kirchoff stress tensor	Pa
t	time	s
T	temperature	$^{\circ}\text{C}$
$\bar{v}$	velocity	m/s
x_a, x_v	mole fraction of air and vapor in gas phase	
V	volume	m^3
x, y, z	directions	m
Greek Symbols		
ρ	density	kg/m^3
λ	latent heat of vaporization	J/kg
ω_a, ω_v	mass fraction of air and vapor	

ϕ	porosity	
μ	shear modulus	Pa
μ_i	dynamic viscosity of phase i	Pa · s
μ_0	permeability of free space	$4\pi \times 10^{-7}$ H/m
ν	Poisson's ratio	
σ	stress	Pa
ε_0	permittivity of free space	8.854×10^{-12} F/m
ε	complex relative permittivity	
ε'	dielectric constant	
ε''	dielectric loss	
α	volume fraction	

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Subscripts

amb	ambient
a, g, s, v, w	air, gas, solid, vapor, water
c	capillary
eff	effective
el	elastic
eq	equilibrium
f	fracture
G	ground (stationary observer)
i	i^{th} phase
M	moisture
0	at time $t = 0$
$surf$	surface

Introduction

Microwave (MW) drying is one of the rapid dehydration techniques that has been successfully applied to dry a variety of hygroscopic and porous materials^{1–8}. Microwave drying of food and agricultural products has been found to result in improved final quality compared with, for example, convective drying^{9–12}. Microwaves heat the material volumetrically causing the temperatures to rise rapidly within the material resulting in internal evaporation of water to vapor. This causes gas pressures to develop within that expel out water from within the core resulting in pressure-driven flow. Consequently, moisture removal is faster and drying times are significantly reduced. However, there are certain limitations of the process, too. Due to microwave absorption, temperatures can rise significantly within the material that may lead to burning¹³. With high temperatures, evaporation rates become higher leading to extremely large internal pressures that may result in explosion and material destruction from within, ultimately resulting in quality loss^{1,14}. Moreover, MW drying results in a non-uniform temperature distribution within the material and in some instances, the edges and corners reach excessively large temperatures resulting in over-drying^{15,16}.

Another important phenomenon that takes place during MW drying is a decrease in volume of the material as moisture is lost during the process¹⁷. This significantly affects the material structure and the way it interacts with the microwaves. The ability of a material to absorb microwaves is a strong function of its dielectric properties, i.e., dielectric constant, ϵ' , and loss factor, ϵ'' . These in turn depend upon the moisture content, material structure and volume. A change in both moisture and volume critically affects the capacity of the material to absorb microwaves^{18,19} that would manifest itself in drying kinetics of the material during MW drying. Based on the above, one observes a complex interaction between electromagnetics, heat and mass transfer and deformation of the material during microwave drying processes.

Therefore, the main purposes of this article are (1) to develop a fundamentals-based framework consisting of coupled electromagnetics, multiphase and multicomponent transport in porous media, and large deformation phenomena, and (2) use the framework to understand drying of a material inside a microwave

cavity. This comprehensive description of the physics of microwave drying by coupling all the relevant phenomena, and in their complete details, will be the novelty of this work. Modeling works in the past have not included all of these physics and their strong coupling—literature modeling studies are detailed in the following paragraph. Using this model, the manuscript sheds light on the effects of some key process parameters of microwave drying on the dried product’s final quality.

Previous mathematical models for microwave drying

The microwave drying process involves a complex coupling of three different physics: volumetric heating due to microwaves, heat, mass and momentum transfer of various species (liquid water, water vapor, air) inside the drying material and large deformations of the material. Additionally, for accurate modeling of electromagnetics, all the different physics need to be implemented in the three-dimensional cavity. Available models to study microwave drying processes to date have not considered all these physics. Most of the models are either one or two dimensional heat and mass transfer models involving effective diffusivity for transport and Lambert’s law for electromagnetics without considering deformation^{20–23}. Effective diffusivity formulations combine all modes of transport together and therefore do not describe the process completely. Moreover, 1D or 2D models may not be directly applicable to 3D and would require significant reformulations. Slightly improved versions include 2D transport models in porous media, empirical models for electromagnetics heating^{24,25} and, in some cases including small deformation analysis involving shrinkage coefficients^{2,26}. Empirical models for electromagnetics do not account for non-uniform heating inside the material and in most cases provide only a qualitative description of the process. Small deformation analysis may be valid in situations where material shrinkage is less than 10%. But, this may not always be true since highly wet materials (e.g., fruits and vegetables) have been found to shrink up to 40% of their volume requiring large deformation analysis²⁷. Moreover, involving shrinkage coefficients lumps all driving forces for deformation into empirical constants without showing the relative importance of the different modes of

deformation. Other approaches involve considering the 3D cavity and solving Maxwell's equations for electromagnetics, heat and mass transfer in the material treated as a porous medium with^{28,29} and without³⁰⁻³² pressure driven flow and internal evaporation but not accounting for material deformation that is so critical for drying processes. Therefore, in this study, we develop a fundamentals-based model that can be applied to processes that include electromagnetic heating, transport phenomena and deformation of the material during the process. The framework includes the solution of Maxwell's equations of electromagnetics in 3D to obtain the electric field inside the oven cavity and the drying sample. A 3D multiphase transport and poromechanics based model is formulated based on conservation laws to describe the heat, mass, momentum transfer and solid mechanical deformation of the porous material. The three different sets of physics are fully coupled to model the microwave drying process. Note here that the framework developed in this work extends beyond microwave drying and is applicable to many other microwave-related processes.

Objectives and organization of the manuscript

The main objectives of this study are to :

- to develop and solve a mechanistic model for microwave drying taking into account the deformation of the material during the drying process;
- to validate the computational model by performing experiments to measure point temperature histories, spatial temperature distribution, moisture loss histories and shrinkage history for the material dried using microwaves; and
- to perform sensitivity analysis with respect to some key parameters used in the computational model.

The manuscript is organized as follows: the mathematical formulation of coupled electromagnetics, multiphase transport in the material treated as a porous media and large deformation model is presented first. Details about experimentation to measure key process parameters is discussed next. The developed model

is validated using the experimental results and its predictions are used to understand the microwave drying process of a hygroscopic porous material and to highlight the role material deformation plays during the microwave drying process. Finally, a parametric sensitivity analysis is carried out for uncertain parameters, for example, intrinsic permeability and elastic modulus of the deforming solid.

Problem description

A schematic of the problem description is shown in Figure 1. Microwave drying of potato samples (a wet porous material) is studied both experimentally and through detailed mathematical modeling. An accurate description of the microwave drying process requires the solution of electromagnetics in the microwave oven cavity and the wet material, heat and moisture transport and large deformations of the material during drying (Figure 2). Moisture loss and internal pressure generation during microwave drying results in material deformation. Due to deformation, the structure (porosity) of the material changes causing liquid and gas permeabilities to change that affect moisture transport within the material. This in turn affects the dielectric (electromagnetic) properties of the material and microwave power absorption making the problem three-way coupled. The three different sets of physics and their coupling are discussed in detail in the following sections.

Electromagnetics in the oven cavity and drying material

Maxwell's equations for electromagnetics of a variable dielectric property are solved to compute the electric field distribution inside the oven cavity and the drying material:

$$\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H} \quad (1)$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon_0\varepsilon\mathbf{E} \quad (2)$$

$$\nabla \cdot \varepsilon\mathbf{E} = 0 \quad (3)$$

$$\nabla \cdot \mathbf{H} = 0 \quad (4)$$

Here, $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field intensity, respectively, and ε_0 is the permittivity of free space. The complex relative permittivity, ε , of the material is defined in terms of dielectric constant, ε' , and dielectric loss, ε'' , as functions of temperature and moisture content obtained as outputs from the transport model described in Section “Multiphase transport in porous media”:

$$\varepsilon = \varepsilon'(T, M) - j\varepsilon''(T, M) \quad (5)$$

Power absorption

Power absorbed per unit volume in the material is obtained from the Poynting Theorem:

$$Q(x, y, z, T) = \frac{1}{2} \omega \varepsilon_0 \varepsilon''(\mathbf{E} \cdot \mathbf{E}^*) \quad (6)$$

Excitation

The excitation for the microwave oven is through a rectangular waveguide with TE_{10} as the dominant mode of the transverse electric (TE) waves. The actual microwave power output for the oven is different from the rated power by the manufacturer and was obtained experimentally according to the procedure described elsewhere³³. The actual value of output power was experimentally measured as 786.8 W and was used as the input power for calculating the power absorbed by the material.

Multiphase transport in porous media

Multiphase, multicomponent transport inside the material treated as a porous medium is described here. The porous medium is considered as a continuum consisting of solid, liquid water and the gas phase that is assumed as a binary mixture of water vapor and air. Mass, momentum and energy conservation equations are described for the different phases undergoing different transport modes that include binary diffusion, capillarity and bulk flow. The effect of microwave heating is included as a source term obtained from

the electromagnetics model. Coupling between electromagnetics and transport inside the food material is carried out via changes in dielectric properties that are functions of temperature and moisture. With changes in dielectric properties, electric field distribution and power dissipated (electromagnetic heating) inside the material changes (see Figure 2).

A schematic of the Representative Elementary Volume (REV) of the material is shown in Figure 3. The material continuum is mathematically expressed in terms of the volume occupied by solid and fluid phases:

$$\Delta V = \Delta V_s + \Delta V_f \quad (7)$$

Inside the REV, fraction of volume occupied by the liquid and gas phase is defined in terms of total porosity (ϕ) as:

$$\phi = \frac{\Delta V_w + \Delta V_g}{\Delta V} = \phi_w + \phi_g \quad (8)$$

The material deforms during drying resulting in a change in its structure (porosity). But, the volume of the solid phase does not change, i.e., it is incompressible. Based on this, the material porosity at any instant of time, t , can be determined by conserving the volume of the solid:

$$\Delta V (1 - \phi) = \Delta V_0 (1 - \phi_0) \quad (9)$$

where, ϕ_0 and ΔV_0 are the initial porosity and initial volume of the REV, respectively.

Momentum conservation

Darcy's law is used to describe the convective transport (or bulk flow) of fluid phases in the porous medium. Bulk flow occurs due to gas pressure gradients that develop due to unequal rates of evaporation within the material:

$$v_{i,s} = -\frac{k_{in,i}k_{r,i}}{\mu_i} \nabla p_i \quad (10)$$

where, $i = w, g$ denotes the liquid water and gas phase, respectively. The gas pressure, p_g , is the sum of partial pressures exerted by water vapor, p_v , and air, p_a , based on ideal gas law and is calculated by solving the overall mass balance equation of the gas phase (Eqn. 12). Since the material deforms, the velocities of the fluid phases, $\bar{v}_{i,s}$, in Eqn. 10 above, are obtained relative to the velocity of the solid phase, $\bar{v}_s$.

Mass conservation

Mass conservation equations for liquid water, gas and water vapor are given by:

$$\frac{\partial c_w}{\partial t} + \bar{\nabla} \cdot \bar{n}_{w,G} = -\dot{I} \quad (11)$$

$$\frac{\partial c_g}{\partial t} + \bar{\nabla} \cdot \bar{n}_{g,G} = \dot{I} \quad (12)$$

$$\frac{\partial(c_g \omega_v)}{\partial t} + \bar{\nabla} \cdot \bar{n}_{v,G} = \dot{I} \quad (13)$$

where, $\dot{I}$ denotes the phase change. Equations 11–13 solve for the respective volumetric concentrations, c_i , that are related to their relative saturations:

$$S_i = \frac{\Delta V_i}{\Delta V_p} = \frac{\Delta V_i}{\phi \Delta V} \quad i = w, g \quad (14)$$

i.e., $c_w = \rho_w S_w \phi$ and $c_g = \rho_g S_g \phi$.

Due to material deformation, the various fluxes in Eqns. 11–13, $(\bar{n}_{i,G})$, needs to be defined with respect to the ground frame of reference. This is obtained by adding an additional flux arising due to movement of the solid phase to the various fluxes obtained with respect to the deforming solid:

$$\underbrace{\bar{n}_{i,G}}_{\text{Flux relative to ground frame}} = \underbrace{\bar{n}_{i,s}}_{\text{Flux relative to deforming solid}} + \underbrace{c_i \bar{v}_s}_{\text{Additional flux due to solid movement}} \quad (15)$$

Mass fluxes

Liquid water flux. The total flux of liquid water phase arises due to the net pressure acting on it. Liquid water inside a pore experiences: 1) gas pressure that drives the water out to regions of low pressure, and 2)

capillary pressure that tries to hold the water within the pore and is attractive in nature. Therefore, the net pressure acting on the liquid phase is given by:

$$p_w = p_g - p_c \quad (16)$$

and the flux of liquid water, $\bar{n}_{w,s}$, relative to the solid phase is given by:

$$\bar{n}_{w,s} = -\rho_w \frac{k_{in,w} k_{r,w}}{\mu_i} \nabla p_w \quad (17)$$

$$= -\rho_w \frac{k_{in,w} k_{r,w}}{\mu_w} \nabla (p_g - p_c) \quad (18)$$

$$= -\rho_w \frac{k_{in,w} k_{r,w}}{\mu_w} \nabla p_g + \rho_w \frac{k_w k_{r,w}}{\mu_w} \nabla p_c \quad (19)$$

$$= \rho_w \bar{v}_{w,s} + \rho_w \frac{k_{in,w} k_{r,w}}{\mu_w} \frac{\partial p_c}{\partial c_w} \nabla c_w \quad (20)$$

where, $\bar{v}_{w,s}$ is substituted from Eqn. 10 above that accounts for bulk flow arising due to gas pressure gradients. The second term in Eqn. 20 arises due to capillarity and is written in terms of capillary diffusivity,

$$D_{w,cap} = -\rho_w \frac{k_{in,w} k_{r,w}}{\mu_w} \frac{\partial p_c}{\partial c_w}, \text{ as:}$$

$$\bar{n}_{w,s} = \rho_w \bar{v}_{w,s} - D_{w,cap} \nabla c_w \quad (21)$$

Gas flux. The net flux for the gas phase with respect to the solid, $\bar{n}_{g,s}$, is based on Darcy's law above and is given by:

$$\bar{n}_{g,s} = -\rho_g \frac{k_g k_{g,r}}{\mu_g} \nabla p_g \quad (22)$$

Water vapor flux. The flux of water vapor relative to the solid phase, $\bar{n}_{v,s}$, arises due to 1) bulk flow given by Darcy's law (Eqn. 10), and 2) binary diffusion with air (Fick's law):

$$\bar{n}_{v,s} = -\rho_v \frac{k_{in,g} k_{g,r}}{\mu_g} \nabla p_g - \left(\frac{C_g^2}{\rho_g} \right) M_v M_a D_{bin} \nabla x_v \quad (23)$$

$$= \rho_v \bar{v}_{g,s} - \left(\frac{C_g^2}{\rho_g} \right) M_v M_a D_{bin} \nabla x_v \quad (24)$$

The gas phase is assumed a binary mixture of water vapor and air with their corresponding concentrations obtained by solving for their respective mass fractions, ω_v and ω_a . The vapor and air concentrations are

related to the gas phase concentration by their mass fractions, i.e., $c_v = \omega_v c_g$ and $c_a = \omega_a c_g$. The mass fraction of vapor is obtained by solving Eqn. 13 and the mass fraction of air is obtained via closure equation:

$$\omega_a = 1 - \omega_v \quad (25)$$

As discussed in Eqn. 15, the net flux of liquid water, gas and water vapor needs to be defined with respect to the ground frame and is given by:

$$\bar{n}_{w,G} = \rho_w \bar{v}_{w,s} - D_{w,cap} \nabla c_w + c_w \bar{v}_s \quad (26)$$

$$\bar{n}_{g,G} = \rho_g \bar{v}_{g,s} + c_g \bar{v}_s \quad (27)$$

$$\bar{n}_{v,G} = \rho_v \bar{v}_{g,s} - \left(\frac{C_g^2}{\rho_g} \right) M_v M_a D_{bin} \nabla x_v + c_v \bar{v}_s \quad (28)$$

Energy conservation

The three phases of the continuum are assumed to be in thermal equilibrium and therefore only one energy balance equation is solved for the mixture. Energy conservation includes convection of fluid phases, heat conduction, evaporative cooling and microwave heat source term computed from Eqn. 6:

$$\rho_{eff} C_{p,eff} \frac{\partial T}{\partial t} + \sum_{i=w,v,a} (\bar{n}_i \nabla (C_{p,i} T)) = \nabla (k_{eff} \nabla T) - \lambda \dot{I} + Q_{mic}(x, y, z, t) \quad (29)$$

The properties of the mixture, ρ_{eff} , $C_{p,eff}$, k_{eff} , are obtained by averaging those of pure components, weighted by either their mass or volume fractions:

$$\rho_{eff} = (1 - \phi) \rho_s + \phi (S_w \rho_w + S_g \rho_g) \quad (30)$$

$$C_{p,eff} = m_s C_{p,s} + m_w (S_w C_{p,w} + m_g C_{p,g}) \quad (31)$$

$$k_{eff} = (1 - \phi) k_s + \phi (S_w k_w + S_g (\omega_v k_v + \omega_a k_a)) \quad (32)$$

Phase change

Internal evaporation of liquid water to vapor results in the development of large internal pressures within the material due to volumetric heating. Phase change is thus spatially distributed throughout the domain. An

explicit expression for phase change, $\dot{I}$, is used in which the rate is proportional to the difference between actual vapor density (obtained as a model output) and equilibrium vapor density obtained from moisture sorption isotherm³⁴:

$$\dot{I} = K_{evap}(\rho_{v,eq} - \rho_v)S_g\phi \quad (33)$$

Equation 33 has been used by others for studying transport with phase change in a hygroscopic material^{35,36} and it is consistent with statistical rate theory based approach for evaporation above pure water surface³⁷.

In Eqn. 33 above, K is the evaporation rate constant and is defined as the reciprocal of the equilibration time for the phase change process i.e. transition of a molecule from liquid water to water vapor and vice versa. For porous materials having an average pore diameter of $25 \mu m$ such as potatoes, the equilibration time has been estimated to be about $10^{-5} s$ ³⁴. This translates to a K value of $100,000 s^{-1}$. However, during numerical implementation, it was found that for $K = 1000 s^{-1}$ and above there was negligible change in the solution, for example, temperature and moisture values. Ideally, a value of $K = 100,000 s^{-1}$ should have been used, but this led to convergence issues of the numerical scheme and increased the computation time significantly due to smaller time steps needed. Since, for $K = 1000 s^{-1}$ and $10,000 s^{-1}$ there was practically no change in the results, it was decided to proceed with a K value of $1000 s^{-1}$ to make it relatively easy on the numerical implementation. Furthermore, times scales for heat and mass transport lie between $10^{-1} s$ to $10^{-3} s$ for the pore sizes of potatoes and are comparatively larger or equal to that of time scale for evaporation justifying the value used. The explicit expression for evaporation rate such as Eqn. 33 above greatly simplifies implementation in a commercial software that typically has standard transport equations built in. A detailed discussion of this non-equilibrium formulation can be found elsewhere³⁴.

Solid mechanics formulation for large deformations

A poromechanics-based approach is followed to model deformation during microwave drying²⁷. Two driving forces for deformation arise: 1) moisture loss during drying, and 2) gradients in gas pressure arising due

to internal evaporation from microwave power absorption. Potatoes were found to shrink by 20% or more of their initial volume through experiments thus necessitating a large deformation analysis.

Deformation gradient tensor

In large deformation analysis, the deformation gradient, $\mathbf{F}$, is used to map the displacements in the reference frame to those in the spatial frame (Figure 4). In Lagrangian description of motion, the coordinates of spatial frame, $\bar{x}$, are a function of the coordinates in the material frame, $\bar{X}$, and current time, t :

$$\bar{x} = \mathbf{X}(\bar{X}, t) \quad (34)$$

The displacement of a material point is defined in terms of a displacement vector, $\bar{u}$, as:

$$\bar{u}(\bar{X}, t) = \mathbf{X}(\bar{X}, t) - \bar{X} \quad (35)$$

The deformation gradient, $\mathbf{F}$, and the Jacobian, J , are then defined as:

$$\mathbf{F} = \bar{\nabla}_X \cdot \bar{x} = \bar{\nabla}_X \cdot (\mathbf{X}(\bar{X}, t)) \quad (36)$$

$$J = \det(\mathbf{F}) \quad (37)$$

Here, J gives the ratio of final to initial volume of the REV at any instant of time, t . From Eqn. 9, the porosity of the material at any instant can be written in terms of J as:

$$\phi(t) = 1 - (1 - \phi_0) \frac{\Delta V_0}{\Delta V} = 1 - \frac{1 - \phi_0}{J} \quad (38)$$

Multiplicative split of the Deformation Gradient Tensor

For large deformation analysis, the deformation gradient, $\mathbf{F}$, is characterized by a multiplicative split of the elastic deformation gradient, $\mathbf{F}_{el}$, and deformation gradient due to moisture effects, $\mathbf{F}_M$ to account for material deformation as shown in Figure 5. The elastic deformation gradient, $\mathbf{F}_{el}$, depends upon the mechanical (elastic) properties of the material, Young's Modulus and Poisson's Ratio. Deformation gradient

associated with moisture, $\mathbf{F}_M$, accounts for volume changes as affected by moisture loss from the material at any given location, i.e., any loss in volume occurring due to moisture removal is taken up by the solid phase—it will be discussed in Section “Constitutive law.” This is similar with finite strain thermo-elasticity where the material deformation is characterized by elastic and thermal effects³⁸:

$$\mathbf{F} = \mathbf{F}_M \mathbf{F}_{el} \quad (39)$$

The above split in deformation gradient is analogous to small deformation analysis in which an additive split of moisture and/or thermal strains from the total strain is used to obtain the mechanical strains in the material that are then related to the stress due to mechanical loads via Hooke’s law^{39–41}.

Solid momentum balance

Solid momentum balance is used to solve for the stresses that develop inside the material during the microwave drying process:

$$\nabla \cdot \boldsymbol{\sigma}'' = \nabla p_g \quad (40)$$

where, $\boldsymbol{\sigma}''$ is the Cauchy stress tensor. The body force term on the right hand side of Eqn. 40 arises due to gradients in gas pressure from internal vapor generation with the application of microwaves²⁹. Since Lagrangian measures of stresses and strains are used for large deformation analysis, Eqn. 40 is written in material coordinates as:

$$\nabla_X \cdot (\mathbf{S}'' \mathbf{F}_{el}^T) = \nabla p_g \quad (41)$$

where, $\mathbf{S}''$ is the second Piola-Kirchhoff (PK2) stress tensor. The PK2 stress, $\mathbf{S}''$, is related to the Cauchy stress, $\boldsymbol{\sigma}''$, by:

$$\mathbf{S}'' = J_{el} \mathbf{F}_{el}^{-1} \boldsymbol{\sigma}'' \mathbf{F}_{el}^{-T} \quad (42)$$

where, J_{el} is the elastic volume ratio of the material given by the determinant of the elastic deformation gradient tensor, $\mathbf{F}_{el}$. The second law of thermodynamics is used to relate the PK2 stress, $\mathbf{S}''$, with Green-

Lagrange tensor for elastic strains, $\mathbf{E}_{el}$, via a strain energy function, W_s :

$$\mathbf{S}'' = \frac{\partial W_s}{\partial \mathbf{E}_{el}} \quad (43)$$

Constitutive Law

A constitutive equation for the elastic strain energy density, W_s , is needed to characterize material deformation. For potatoes, material constitutive law has been assumed to follow Hooke's law for strain levels beyond 10% which may not be valid⁴¹. Stress-strain curves for potatoes obtained in compression at different moisture contents suggest a non-linear elastic or a hyperelastic behavior⁴². Based on this, a modified Neo-Hookean constitutive model is chosen to characterize material deformation during microwave drying⁴³:

$$W_s = \frac{K}{2} (J_{el} - 1)^2 - \frac{\mu}{2} (\bar{I}_1 - 3) \quad (44)$$

In Eqn. 44, K is the bulk modulus and μ is the shear modulus of the material that are related to the elastic modulus, E , and Poisson's ratio, ν . $\bar{I}$ is the first invariant of the right-Cauchy Green tensor, $\bar{\mathbf{C}} (= \bar{\mathbf{F}}_{el}^T \bar{\mathbf{F}}_{el})$ for the volume preserving (deviatoric) part of the elastic deformation tensor, $\bar{\mathbf{F}}_{el}$. The deviatoric part of the deformation gradient, $\bar{\mathbf{F}}_{el}$, is related to the dilatation part, $J_{el}^{1/3}$, and the deformation gradient, $\mathbf{F}_{el}$, as⁴⁴

$$\mathbf{F}_{el} = J_{el}^{1/3} \bar{\mathbf{F}}_{el} \quad (45)$$

The elastic Jacobian, $J_{el} = \det(\mathbf{F}_{el})$, is obtained as the ratio of total volume change, J , and volume changes due to moisture effects, J_M (Eqn. 46). With the elastic deformation, $\mathbf{F}_{el}$ accounted for in Eqn. 45 above, the deformation gradient due to moisture loss, $\mathbf{F}_M = J_M \mathbf{I}$, is needed next. Potato tissue is initially saturated with water and is soft and rubbery. Their volumetric shrinkage during drying in the rubbery regime occurs primarily due to moisture loss⁴⁵. The material becomes hard and glassy at very low moisture contents (≈ 0.3 dry basis) during which deviations in volumetric shrinkage is observed⁴⁶. The final moisture content of potato samples obtained from microwave drying experiments (3.3 dry basis) confirmed that the material stayed in the rubbery region for the entire period of drying. The volume change due to moisture loss, J_M ,

is obtained by changes in the volume of liquid water within the material at any given location i.e. any loss in volume occurring due to moisture removal is taken up by the solid phase. This is achieved by conserving the volume of the solid phase which does not change during the drying process. Therefore, J_M can be calculated by relating it to the change in the volume fraction, ϕ_w , of liquid water inside the REV^{27,47}:

$$\Delta V(1 - \phi_w) = \Delta V_0(1 - \phi_{w,0}) \quad (46)$$

$$\frac{\Delta V}{\Delta V_0} = J_M = \frac{1 - \phi_{w,0}}{1 - \phi_w} \quad (47)$$

Initial and boundary conditions

The initial conditions for all the variables are listed in Table 1. Boundary conditions for the three different sets of physics are shown in Figure 1 and are listed below.

Electromagnetics

Boundary conditions are specified on the walls of the microwave cavity and along the symmetry plane. The walls are assumed to be perfect electrical conductors in which the electric field is zero. At the air-wall interface inside the oven cavity, the tangential component of electric field is zero:

$$E_{\text{tangential, wall}} = 0 \quad (48)$$

For the symmetry plane, Perfect Magnetic Conductor boundary condition is provided that imposes a symmetry for electric fields and electric currents.

Multiphase transport in porous media

The material inside the microwave oven is at ambient conditions. Pressure at the open surface of the drying sample is set to ambient pressure (1 atm):

$$p_{\text{surf}} = p_{\text{amb}} \quad (49)$$

Liquid water moves from within the material and reaches the surface where it is convected away as vapor. In case the material surface becomes saturated (i.e., $S_w=1$), water can move out directly by drip flow. The relative humidity inside the microwave oven was measured during microwave drying experiments to obtain the water vapor concentration, $\rho_{v,oven}$ in the surrounding air, and was found to be 30%:

$$\bar{n}_{w,s}|_{surf} = h_m \phi S_w (\rho_v - \rho_{v,oven}) + \underbrace{c_w \bar{v}_{n,w}}_{\text{when } S_w=1} \quad (50)$$

The flux of liquid water at the surface (Eqn. 50) is defined with respect to the deforming solid. In addition to water lost through surface evaporation, moisture flux at the boundary also moves along with the movement of the surface due to deformation. Therefore, the total moisture lost from the surface with respect to the ground frame is given by:

$$\bar{n}_{w,G}|_{surf} = \bar{n}_{w,s}|_{surf} + c_w \bar{v}_s \quad (51)$$

Water vapor reaches the surface via bulk flow and binary diffusion and is convected away at the surface. In case of large pressure gradients, it may undergo blowing near the surface. As with liquid water flux, the vapor flux at the boundary is updated to account for the deforming surface:

$$\bar{n}_{v,s}|_{surf} = h_m \phi S_g (\rho_v - \rho_{v,oven}) + \underbrace{c_v \bar{v}_{n,v}}_{\text{blowing}} \quad (52)$$

$$\bar{n}_{v,G}|_{surf} = \bar{n}_{v,s}|_{surf} + c_v \bar{v}_s \quad (53)$$

For energy equation (Eqn. 29), forced convective heat transfer takes place at the boundary:

$$q|_{surf} = h_t (T_{oven} - T_{surf}) - \lambda \bar{n}_{w,G}|_{surf} - \sum_{i=w,v} (\bar{n}_{i,G}|_{surf}) C_{p,v} T_{surf} \cdot \bar{N}_{surf} - \underbrace{c_w \bar{v}_{n,w} C_{p,w} T}_{\text{when } S_w=1} \quad (54)$$

The first term on the right hand side of Eqn. 54 is the convective heat transferred from the surrounding air to the open surface of the material. The average temperature of the surrounding air, measured by placing a thermocouple inside the oven cavity during the drying process, was found to be 33°C. The second term is the heat lost due to surface evaporation of liquid water, the third term is the energy lost due to bulk flow of water vapor normal to the surface ($\bar{N}_{surf}$ is the unit surface normal), and the fourth term is the energy lost due to drip flow of water.

Solid mechanics

The displacements normal to the symmetry surface are set to zero while the open surfaces are free to deform and have no constraints. For the part of the material in contact with the string (see Figure 1), displacements are allowed only along the string while those normal to the string are set to zero.

Input parameters

Input parameters required for simulating the microwave drying process is summarized in Table 1. Details about some of the important parameters are discussed here.

Transport properties

Transport of liquid water due to capillary effects is modeled as diffusion with capillary diffusivity of liquid water inside potatoes defined as function of dry basis moisture content, M (Eqn. 55) ⁽⁴⁸⁾. Binary diffusivity of water vapor in air is defined in terms of total porosity of the drying material and the gas phase saturation (Eqn. 56) ⁽⁴⁹⁾

$$D_{w,cap} = 1 \times 10^{-8} \exp(-2.8 + 2.0M) \quad (55)$$

$$D_{bin} = 2.6 \times 10^{-5} (S_g \phi)^{3-\phi} / \phi \quad (56)$$

There is typically a gentle forced convective air flow inside the microwave oven that produces a small value of heat transfer coefficient above natural convection ⁽⁵⁰⁾. This value has minor spatial variation along the flow, as expected. For the current study, a representative constant value of 10 W/m²K was used based on values reported in literature ^(29,50). The Chilton-Colburn analogy was used to calculate the convective mass transfer coefficient ⁽⁵¹⁾

$$h_m = \frac{h_t D_{bin} Le^{0.33}}{k_v} \quad (57)$$

$$Le = \frac{k_v}{\rho_g C_{p,g} D_{bin}} \quad (58)$$

For intrinsic permeability of liquid water, a value of $1 \times 10^{-15} \text{ m}^2$ was used^{48,52-54}. A sensitivity analysis follows in Section “Sensitivity analysis” since measurements of intrinsic permeability for food systems is difficult requiring extremely large pressures to pump the fluid across the food material⁵⁵. Intrinsic permeability for the gas phase is different from that of liquid water and is calculated from liquid water permeability via the Klinkenberg correction factor (Eqn. 59)⁵⁶. For extremely small pore sizes, such as potatoes ($0.542 - 3.88 \mu\text{m}$ ⁵⁵, the gas flow gets enhanced due to slip flow of the gas phase at pore walls and therefore necessitates to account for the Klinkenberg effect⁵⁶.

$$k_{in,w} \left(1 + \frac{0.15 k_{in,w}^{-0.37}}{p} \right) \quad (59)$$

The relative permeabilities of the gas and liquid phase obtained as functions of their relative saturations, S_w and S_g , defined as⁵⁷:

$$k_w^r = \begin{cases} \left(\frac{S_w - 0.09}{0.91} \right)^3 & S_w > 0.09 \\ 0 & S_w < 0.09 \end{cases} \quad (60)$$

$$k_g^r = \begin{cases} 1 - 1.1 S_w & S_w < 0.91 \\ 0 & S_w > 0.91 \end{cases} \quad (61)$$

During microwave drying, the material undergoes large deformations resulting in significant changes in the porosity of the material (Eqn. 9). As ϕ changes, it alters the resistance to flow of fluid phases through the material. This is accounted for by changing the liquid and gas permeabilities by multiplying a porosity factor given by the Kozeny-Carman equation⁵⁷:

$$f(\phi) = \left(\frac{\phi}{\phi_0} \right)^3 \left(\frac{1 - \phi_0}{1 - \phi} \right)^2 \quad (62)$$

Dielectric properties

In order to compute the microwave power absorbed by the material during drying, dielectric properties as functions of temperature and moisture are needed. As the material dries, it loses moisture, undergoes

shrinkage and its temperature increases, affecting its dielectric properties significantly. Several measurement techniques have been used to measure the dielectric properties of porous materials but without much success as there have been reports of poor repeatability^{58,59}. Therefore, the other alternative is to estimate them using mixture average equations^{24,31,60,61}. The two most commonly used equations to predict dielectric properties of mixtures from properties of individual components are the Complex Refractive Index mixture equation (CRIME) and Landau and Lifshitz, Looyenga equation (LLLE). For food materials, the Landau and Lifshitz, Looyenga equation⁶² has been used to predict the dielectric properties of cereal grains⁶³ and bread⁶¹ for a range of microwave frequencies with sufficient accuracy. Therefore, for the present work, the LLLE equation was used to predict changes in dielectric properties of potatoes based on its moisture loss and deformation characteristics during the drying process:

$$\varepsilon^{\frac{1}{3}} = \sum_{i=s,w,g} \alpha_i \varepsilon_i^{\frac{1}{3}} \quad (63)$$

where, α_i and ε_i are the volume fractions and dielectric properties of phase i , respectively. The volume fraction of solid, liquid water and gas can be written in terms of total porosity, ϕ , and relative saturations of the fluid phases, S_g and S_w , defined above:

$$\alpha_s = 1 - \phi; \quad \alpha_w = S_w \phi; \quad \alpha_g = S_g \phi \quad (64)$$

The relative saturations, S_i , of fluid phases evolve due to phase change and moisture loss from the material and the porosity, ϕ , changes due to material deformation. Dielectric properties of the individual components needed to compute the homogenized property of the material were obtained from literature and are summarized in Table 1.

Mechanical properties

For the strain energy function defined by Eqn. 44, bulk modulus, K , and shear modulus, μ , are needed. These in turn are related to Poisson's ratio, ν , and elastic modulus, E , of the material.

Poisson's Ratio, ν : Poisson's ratio of microwave-treated potatoes has not been reported in literature. However, ν of parenchyma isolated from potato tubers have been measured and found to lie between 0.49 and 0.50⁶⁴. This indicates a rubber-like behavior i.e. when stretched in one direction, the material contracts in the other direction in a manner to conserve the total elastic volume. Also, parenchymatous plant tissues (e.g. potatoes) are composed of cells having almost equal diameter in all directions (isodiametric) whose Poisson's ratio has been estimated as 0.5⁶⁵. Based on this, since the potatoes stay soft and rubbery during the microwave drying process (a change from 4.57 to 3.3 dry basis), a value of 0.49 was estimated and used for the the Poisson's ratio of potatoes during microwave drying.

Elastic Modulus, E : The elastic modulus, E , has been measured as a function of moisture content, M (dry basis), and is given by the following⁶⁷:

$$E(M) = 1.69 \times 10^7 \exp(-0.522M) \quad M \geq 1.0 \text{ (dry basis)} \quad (65)$$

Bulk modulus and shear modulus: Bulk and shear modulus values are obtained in terms of E and ν through standard equations available from solid mechanics literature:

$$K = \frac{E}{3(1 - 2\nu)} \quad (66)$$

$$\mu = \frac{E}{2(1 + \nu)} \quad (67)$$

Poisson's ratio and elastic modulus values were used as inputs to compute bulk and shear modulus values for the strain energy function following Eqns. 66 and 67.

Model Implementation

Figure 1 shows the computational domain along with the different physics being solved. Equations for electromagnetics, multiphase transport and large deformation mechanics are fully coupled. The implementation

of the three different sets of physics in a simulation environment is not straightforward, especially with the addition of solid mechanics, and needs further discussion.

Arbitrary-Eulerian-Lagrangian framework for mesh movement: When the material is dried inside the oven cavity, it shrinks. As a consequence, the air surrounding the material surface occupies the volume that is lost due to shrinkage. In order to account for this phenomenon during computations and to correctly implement the electromagnetics inside the 3D oven cavity, the boundaries of the cavity surrounding the material also need to be moved following material deformation (Figure 6a). Otherwise, in the small region surrounding the material in which deformation occurs, there would be no Degrees of Freedom (DoF) to solve for the electromagnetics variables in the oven domain. This is depicted in Figure 6b that shows the narrow region outside the deformed material where the DoF for electromagnetics in the oven cavity would be zero leading to a failed solution. Note here that these boundaries are part of the microwave cavity and not the material as shown in Figure 6a. In order to account for mesh movement for the oven cavity, an Arbitrary-Lagrangian-Eulerian (ALE) framework is used. It is a powerful tool for Fluid-Structure-Interaction (FSI) type of problems, such as the one considered in this work, as it can account for mesh movement of the ‘fluid’ domains that are connected to the ‘structure’ domains in which deformations occur^{29,71,72}. However, applying the framework for the entire cavity would result in an extremely large number of DoFs making the problem incomputable. Instead, a slightly larger domain of the same shape as the drying material is created on the outside as shown in Figure 6c. This relatively smaller domain contains the fluid, i.e., air and the ALE framework is applied to this domain, as opposed to the entire cavity, bringing in a significant reduction in the number of DoFs. The ALE method for mesh movement moves the inner boundaries of the fluid domain based on the deformation of the drying material while the outer boundaries remain fixed (Figure 6d). This novel methodology for implementing the three-way coupled framework for electromagnetics, transport and mechanics in a 3D environment opens up possibilities to model and understand microwave related processes

which otherwise is implemented via simplistic 2D models.

Solution methodology: The governing equations for electromagnetics, multiphase transport in porous media, solid mechanics for large deformations and ALE framework for mesh movement were solved using the commercially available Finite Element software COMSOL Multiphysics 4.3b (COMSOL, Burlington, MA). The authors research group has successfully implemented various combinations of transport, solid mechanics and electromagnetics problems in this software and verified against experimental data^{27,29,34}. Maxwell's equations for electromagnetics were solved using the RF Module using the GMRES iterative solver with the Geometric Multigrid preconditioner. Mass, momentum and energy conservation for various species were solved using the General form of PDEs, Transport of Concentrated Species, Darcy's Law and Heat Transfer in Fluids modules, respectively, using the MUMPS direct solver. Solid momentum balance was solved using the Non-linear Structural Mechanics module using the modified Neo-Hookean constitutive law for the deforming solid, and ALE framework for mesh movement was solved using the Fluid Structure Interaction module. Tetrahedral mesh elements were used (Figure 7) and mesh size near the material surface was kept half of what was used in the interior (0.6 mm) to help convergence with large pressure gradients. A mesh-independence study was conducted to ensure that any dependent variable did not change by more than 1% of the total change when the element size was reduced by half. Duty cycling at 10% power level (microwave ON for 2 s and OFF for 20 s for a total cycle time of 22 s) was simulated for a total of 28 cycles (≈ 10 minutes) of microwave drying. For every cycle of drying, electromagnetics was solved in the cavity and the material system to obtain the microwave energy absorbed in the material. This was followed by solution of the heat and mass transport and solid mechanics equations in the material using microwave energy as a source term to compute temperature, moisture, pressure and deformation with a maximum time step for computations set as 0.125 s. The procedure was repeated using updated dielectric properties and material size after every cycle. The total number of degrees of freedom to be solved for were $\sim 8,540,000$. The

simulations were run on a 12 core, 2.0 GHz Windows workstation with 32 GB memory for a total run time of 144 hours.

Experimental methodology

Sample preparation

Fresh russet Idaho potatoes procured from a local supermarket (Wegmans, Ithaca, NY) were used for microwave drying experiments. They were stored at room temperature ($25 \pm 0.5^\circ\text{C}$) for 24 h before conducting experiments. Before experimentation, the potatoes were peeled, washed and cut into 1 cm cubes. In order to prevent moisture loss and enzymatic browning, the samples were wrapped using a saran wrap. The initial moisture content of potato samples (an average of three replicates) was gravimetrically measured as 4.577 (dry basis).

Experimental setup

The experimental setup is shown in Figure 8. A domestic microwave (950 W, 2450 MHz, GE, Inc., Louisville, KY, Model number JES738WJ02) having different power level settings (10–100%) was used for conducting the drying experiments. One hole was drilled on the top (1.7 cm diameter) face of the microwave oven to hang the potato samples during drying, measure instantaneous moisture loss and point temperature histories. One hole was drilled on the side (2 cm diameter) to measure instantaneous surface temperature profiles and surface shrinkage of the material. The size of the top hole was made such that any microwave leakage that occurred was below the threshold limit of 5 mW/cm^2 verified using a microwave leakage detector. The hole on the side face was large enough to focus on the potato surface inside the oven and measure surface temperature and shrinkage (discussed later). A cut-off waveguide was inserted to the side hole to prevent microwave leakage. Microwave drying experiments were carried out at the 10% power level setting for a total of 10 minutes of drying.

Moisture loss, temperature and shrinkage histories

Moisture loss measurements. Moisture loss history during drying experiments was measured using a digital weighing balance (SP202, Ohaus Corporation, Pine Brook, NJ; 0.01g least count) connected to a computer. A teflon stick was attached to a hook system located at the bottom of the weighing balance. The teflon stick was passed through the top hole to hold the sample during drying. Due to its low dielectric property ($2 - j0.0008$)⁷³, Teflon does not alter the electric field distribution inside the oven cavity. The drying material was tied to the teflon stick in a manner to avoid sample rotation during the experiment (Figure 8b). Weight loss from the sample during drying was recorded every 5 second interval using a digital software WinWedge 3.4.3 (Taltech, PA). Three replicates were carried out and mean and standard deviation values are reported.

Temperature measurements. During drying, temperature of the sample was measured in terms of 1) spatial profile, and 2) instantaneous point history. Spatial temperature profiles were measured using an infrared thermal imaging camera having an accuracy of ± 2 °C (FLIR A320, FLIR Systems, Boston, MA) that was placed behind the cut-off waveguide on the side face of the microwave oven (Figure 8). FLIR R&D software (version 3.4.13) was used to record and analyze surface temperature profiles. The infrared camera was calibrated using boiling water (100°C) and ice-water mixture (0 °C) as standards. This required changing the emissivity value of the camera software from the default 0.95 to 0.89 following which surface temperature measurements were carried out on the samples. For measuring point temperature data, optical fiber thermometer (FISO FOT- NS-967A, FISO Technologies, Quebec, Canada) was inserted and placed near the geometric center of the sample. The thermometer having an accuracy of ± 0.5 °C was connected to a FISO OSR-4 workstation and measurements were carried out at every 0.3 second interval. The data was analysed using a software, FISO Command WorkStation (v1.10.9; FISO Technologies, Quebec, Canada). Three replicates were carried out and mean values are reported.

Shrinkage measurements Shrinkage during microwave drying was obtained by measuring changes in the surface area of the drying material. Snapshots of the sample obtained using a digital camcorder placed near the hole on the side face were analysed using the software ImageJ (<http://rsbweb.nih.gov/ij>). The measurement procedure involved image segmentation by converting color images to grayscale followed by conversion to binary images in order for the software to identify the surface area for measurements. Three replicates were carried out and mean and standard deviation values are reported.

Results and Discussion

Experimental validation is presented first in terms of key process parameters: point and surface temperatures, moisture loss and shrinkage. This is followed by a comprehensive description of microwave power absorption, temperature, moisture, pressure, evaporation rate and stress development in the material to understand the microwave drying process. Finally, sensitivity analysis is carried out of some key transport and solid-mechanical parameters on moisture loss, shrinkage and stress development during the microwave drying process.

Temperature histories

Computed and measured temperature histories near the center of the material show good agreement (Figure 9). The slight discrepancy between computed and measured values are due to difficulty in placing the fiber optic probe at the exact center of the small sample, thickness of the temperature probe (1.4 mm), and a relatively large response time (~ 1 s) of the temperature probe. Temperature history curve is wavy due to power cycling of the microwaves (magnetron is ON for 2 s and OFF for the next 20 s for a total cycle time of 22 s). The temperatures rise rapidly for the first few minutes due to microwave absorption. Following an initial rapid rise, the temperature stays fairly constant due to an increase in evaporative cooling followed by an eventual decrease in temperature due to a decrease in microwave absorption. The ability to absorb mi-

crowaves decreases due to a decrease in the effective permittivity and the volume of the material as moisture is lost during the drying process as shown in Figures 14 and 16, respectively.

Figure 9 also contrasts temperature history that would be obtained if material shrinkage were ignored. Temperatures are higher for a shrinking material in contrast with if the material did not shrink. In a shrinking material, the solid, to some extent, compensates for the volume of water lost allowing very little gas phase to enter the pores (Figure 15). In a rigid material, volume stays fixed resulting in a relatively higher gas phase saturation. Consequently, the mixture averaged effective permittivity (both dielectric constant and loss factor) is lower for a rigid material in contrast with a material that shrinks (Figure 14). Therefore, microwave energy absorption per unit volume basis is lower for a rigid material resulting in lower temperature values when compared with a shrinking material (more discussion in Section “Change in microwave power absorption during drying” below). As shrinkage during microwave drying is included in the model using solid mechanics, improvements between computed and measured temperature values is seen showing the critical need for inclusion of material deformation in modeling a microwave drying process for biomaterials.

Spatial distribution of temperature at the sample surface Figure 10 shows good agreement between experimentally measured and computed surface temperature profiles after several cycles of microwave drying. Surface temperature is higher during the ON phase of the cycle due to microwave absorption. When the microwave is OFF, temperatures decrease and become almost uniform due to thermal conduction. Some discrepancy is observed between measured temperatures and computed values at certain instants. This is because, for a microwave operating at 10% power level, the microwave should ideally be ON 2s and be OFF for 20s for a total cycle time of 22s. However, in actual experiments the microwave was found to ON for greater than 2s period i.e. between between 2s and 3s. This additional heating time resulted in slightly higher experimental temperatures. However, for modeling, the ON and OFF phases were clearly defined as 2s and 20s respectively and therefore some discrepancy is observed between experimental and

500 computed profiles. Some edge and corner heating is also observed in experiments (see Figure 10 at cycle
501 24) as predicted by the model, as has been reported in literature during microwave heating of cube shaped
502 samples³¹. Note here that the experimental surface temperature profiles are symmetric with respect to the
503 XZ-plane (Figure 1), justifying the simplification of the computational domain by considering only half the
504 oven cavity and substantially reducing the number of degrees of freedom. This is because, for a microwave
505 operating at 10% power level, the microwave should ideally be ON 2s and be OFF for 20s for a total cycle
506 time of 22s. However, in actual experiments the microwave is ON for anywhere between 2.5s and 3s. There
507 is additional time for which the microwave is ON and this is beyond user control. However, for modeling,
508 the ON and OFF phases are clearly defined as 2s and 20s respectively and therefore we have some discrepancy
509 between experimental and computed profiles.

510 **Moisture loss histories**

511 Figure 11 shows good agreement between computed and measured changes in moisture content for the
512 material during drying. The computed moisture loss is slightly lower between 2 and 6 minutes probably
513 because of a slightly more internal evaporation due to microwave power absorption than computed by the
514 model. Figure 11 also contrasts the moisture loss that would be expected if the material did not shrink.
515 As was discussed above, the decrease in dielectric property is less for a deformable material compared
516 to a rigid one that would result in higher microwave absorption and thus higher moisture loss from the
517 material. Also, for a shrinkable material, the length scale for moisture transport via pressure driven flow and
518 capillary diffusion is reduced, contributing to more moisture loss. The model predicts the transient moisture
519 loss reasonably well showing the effectiveness of the coupled multiphase transport and solid mechanical
520 deformation framework applied to microwave drying of hygroscopic porous materials.

Volume and shape changes during microwave drying

Computed and measured changes in the side face surface area of the sample as a function of dry basis moisture content in Figure 12 shows good agreement. Figures 13a and 13b show the corresponding change in total volume and computed displacement of the material as function of moisture content, respectively. Volume changes within the material is brought about by three different factors: 1) change in elastic volume of the material characterized by J_{el} , 2) volume changes resulting due to moisture loss, J_M and, 3) change in volume due to body force term (gas pressure generation) in the solid momentum balance equation (Eqn. 40). For a material having a Poisson's ratio close to 0.5 such as the one considered in this work, there is no change in the elastic volume of the material¹⁴ and therefore, J_{el} stays close to 1. Volume changes resulting from moisture loss and gas pressure gradient is compared with the total volume of the material, J (Figure 13a). As can be seen, there is no appreciable change in volume arising due to gas pressure gradients probably because internal pressure development is not high enough to deform the material owing to its relatively higher bulk modulus. The computed pressure is only slightly above atmospheric (more discussion in Section "Spatial distributions of microwave power absorption, temperature, liquid water concentration" below) contributing negligibly towards volume changes during the microwave drying process. This is consistent with reported work for microwave puffing where much higher pressures (≈ 3 atm gauge) are required to achieve even a 15% increase in volume²⁹. Volume changes due to gas pressure can contribute perhaps in more intense microwave drying processes or for a material having a relatively lower bulk modulus. For this work, volume changes due to moisture loss contributes primarily to material deformation.

Change in microwave power absorption during drying

Figure 14 contrasts the variations of dielectric properties (computed using Eqn. 63) when deformation is included versus if the material was considered rigid. For the same average moisture content value, both dielectric constant and loss factor are lower for a material whose volume stays fixed. During MW drying,

moisture loss and internal evaporation result in a progressive development of the gas phase within the material that does not absorb any microwaves. For a rigid material, the development of gas phase results from both moisture loss and internal evaporation. In contrast for a deformable material, volumetric shrinkage compensates for the amount of water lost and the gas phase within the material is lowered. Figure 15 compares the development of gas porosity, ϕ_g , within the material considering with and without shrinkage. Gas porosity is lower for the shrinking material since shrinkage is almost equal to the amount of water lost as seen in Figure 13a. As a consequence, the pores remain almost saturated with water with very little gas phase present in them. Therefore, at any given location inside the shrinking material, the volume fraction of liquid water stays relatively high contributing towards higher averaged dielectric property of the material compared to if shrinkage were ignored. The wavy nature of the curves arise due to temperature dependency of dielectric properties.

Figure 16 contrasts the power absorbed when shrinkage is considered with the case of a rigid material. As is expected, the rigid material absorbs less power due to a relatively higher decrease in dielectric properties as discussed in Sections “Temperature histories” and “Moisture histories” above. Power absorbed by the material also depends upon its volume. Even though the material shrinks during MW drying, a relatively higher dielectric property value compensates for the loss in volume and the net power absorbed by the shrinking material stays higher than that of a rigid material as shown in Figure 16 manifesting itself in higher temperature and more moisture loss from the material.

As shown above, model predictions compare favorably with experimentally measured values for temperature, moisture and volumetric shrinkage of the material. The need for deformation analysis during microwave drying cannot be over-emphasized since there are significant improvements in model predictions when deformation is considered. Material deformation plays a strong role in the development of material structure which critically affects the dielectric properties of the material. With the model validated for key process parameters and their histories, we now present additional results followed by sensitivity analysis

with respect to some key parameters.

Spatial distributions of microwave power absorption, temperature, liquid water concentration

Microwave power absorption, temperature, pressure, evaporation rate and liquid water concentration profiles at different sections within the material are shown in Figures 17 - 21. Microwave power absorption at two different microwave cycles are shown in Figure 17. Most of the microwave energy is absorbed near the surface facing the port (source for microwave energy) and the magnitude of power absorbed decreases with time due to moisture loss – the spatial distribution however stays the same.

Absorption of microwave power increases temperatures within the material (Figure 18). When the microwave is ON (for 2 s), temperatures increase inside and, the regions of high temperature correspond with the regions of high microwave absorption. When the microwave is OFF, heat transfers to the colder regions within the material making the temperature distribution more uniform. With time, the peak temperatures decrease due to lowered microwave absorption, as discussed in the above section.

High temperatures within the material cause evaporation of liquid water to vapor resulting in the development of internal pressures. The regions of high evaporation and pressure correspond with the regions of high temperatures as shown in Figures 19 and 20. Development of gas pressure increases the state of stress within the material that would expand it (see Section below).

Figure 21 shows the spatial distribution of liquid water concentration for different microwave cycles. Regions of low concentration are found to occur near the center of the material where the temperatures are high due to evaporation when the microwave is ON. Moisture transport from the core to the surface occurs via pressure-driven flow and capillary diffusion when the microwave is OFF. Moisture reaches the surface where it is convected away as vapor. Gradients in liquid water concentration between the core and the surface reduce when the microwave is OFF resulting in a fairly uniform profile throughout the material.

This results in uniform drying that helps to preserve the overall quality of the material.

Stress development during microwave drying

The addition of solid mechanics into the modeling framework not only allows us to accurately predict key process parameters, for example, temperature and moisture, but also the stress state within the material dried using microwaves, thus predicting material damage and failure. Generally speaking, since microwaves heat more volumetrically in contrast with hot air heating that is mostly near the surface, higher stresses develop near the core as opposed to the material surface. Higher stresses develop near regions of high temperature, moisture and pressure gradients, and if the stress exceeds the failure stress, crack will initiate. For microwave-drying processes, material failure (or stress cracking) may occur inside the material as opposed to near the surface¹. Stress cracking is an undesirable quality attribute and negatively impacts the final quality of the dried product. In order to completely characterize the stress state of the material and to predict material failure, effective stress in terms of components of stress in normal (σ_{xx} , σ_{yy} , σ_{zz}) and shear stresses (σ_{xy} , σ_{yz} , σ_{xz}) is needed. The effective stress, σ_{eff} , within the material is defined in terms of the six stress components given by the von-Mises stress:

$$\sigma_{eff} = \frac{1}{2} \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{yy} - \sigma_{zz})^2 + (\sigma_{xx} - \sigma_{zz})^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2)} \quad (68)$$

Figure 22 shows the computed effective stress within the material during the MW drying process. Stress values are relatively higher in regions with relatively larger gradients in temperature and moisture (facing the port of the microwave cavity). Stress development is contributed by both shrinkage from moisture loss and gas pressure generation. The regions of higher effective stress values correspond with those of higher gas pressure gradients shown in Figure 23. When the microwave is ON, the stress magnitudes and gas pressure gradients are higher and, when pressure gradients subside during the OFF phase, the stress levels also decrease. Thus, for realistic predictions of stress development during microwave drying, it is equally important to consider the gas pressure generation within the material – stress development based on

shrinkage due to moisture loss alone may underestimate the magnitude of stress.

Effective stress and failure stress during microwave drying

Effective stress as opposed to maximum principle tensile stress is used as the failure criterion for materials in their rubbery state since they undergo plastic deformations first before fracturing³⁹. For the range of moisture contents considered in this work, the material stays in the rubbery regime and hence the effective stress criterion is used to predict material failure. Figure 24 shows the evolution of maximum effective stress within the material as a function of average moisture content along with the corresponding failure stress of the material⁶⁷. The stress curve is wavy in nature because of an increase in gas pressure when the microwave is ON (Figure 20) and decrease when the pressure decreases during the OFF phase. The effective stress increases with decreasing moisture contents due to an increase in strain levels when the material shrinks due to moisture loss. Note that the effective stress stays well below the fracture stress at any given moisture and does not lead to any crack initiation. This was experimentally validated as there was no sign of crack development or fissuring within the material.

Sensitivity analysis

Table 1 lists a large number of input parameters that are required to accurately model the microwave drying process. Given the unavailability and variability of some of the parameters, a parametric sensitivity analysis is carried out in order to have greater confidence in the model predictions. It is also necessary to understand the need for precise measurements of certain parameters for which experimental measurements may be difficult. For example, there is large variability in the value of intrinsic permeability and elastic modulus values for potatoes. To determine how these affect moisture loss and material deformation during microwave drying, sensitivity analysis was carried out for the two parameters. Elastic modulus and intrinsic permeability values were varied by $\pm 20\%$ and $\pm 50\%$ from the reference case to include a large variation

based on reported values in literature^{29,42}. Moisture content and volume shrinkage histories were computed as affected by the variations in the elastic modulus and permeability values.

Intrinsic permeability

Intrinsic permeability of 10^{-15} m^2 was used for the reference case⁵⁴ and this value was varied by $\pm 50\%$ for sensitivity analysis²⁹. Moisture loss histories for the different permeability values are shown in Figure 25a, showing no appreciable difference for this range of permeability values. This is probably because capillary diffusion dominates over pressure driven flow for MW drying. This is expected since computed internal pressures are only slightly above atmospheric values resulting in a lower convective flux—such lower pressures result from lower power levels as would be typical in a drying (as opposed to reheating) situation. Figure 25b compares the sensitivity of volumetric shrinkage to permeability values. As with moisture, volumetric shrinkage of the material is not affected as much and shrinkage due to moisture loss dominates over expansion due to gas pressure gradients. The shrinkage history curves for different permeability values are almost coincident with the reference case. With a lower permeability value, there is additional resistance to the flow of fluid phases resulting in an increase in gas pressure within the pore. However, the increase in pressures do not cause any expansion to alter the material's structure that can affect moisture transport significantly. Based on this, moisture loss and volumetric shrinkage are not as sensitive to intrinsic permeability values and therefore their precise values are less critical for model development.

Elastic modulus

Elastic modulus of potatoes as a function of moisture content has been measured as discussed in Section . For the constitutive equation, shear modulus values are needed that can be calculated if the elastic modulus of the material is known (Eqn. 67). For a material having a lower shear modulus, material fracture can occur with relative ease. The elastic modulus value was varied within the range $\pm 20\%$ of the reference case based

on values reported in literature^{42,67}. Figures 25c and 25d show the moisture loss and volumetric shrinkage histories are not sensitive to the variations in elastic modulus for the range considered. However, increasing elastic modulus increases the shear modulus of the material thus increasing the effective stress, σ_{eff} , within the material (Figure 26). For the extent of variation in elastic modulus considered, the effective stress stays lower than the failure stress of the material. Changes in elastic modulus values do not affect shrinkage and moisture loss histories. However, its accurate values are needed to predict material failure that may lead to quality loss during MW drying processes.

To summarize, the choice of intrinsic permeability and elastic modulus do not affect moisture loss and volumetric shrinkage of the material during microwave drying as much while the stress state within the material is highly sensitive to elastic modulus values and therefore requires precise measurements.

Conclusions

A three-way coupled model for electromagnetics in the oven cavity, multiphase transport and large deformation during MW drying of a hygroscopic porous material was developed and solved using finite elements. A novel implementation methodology of the coupled framework was presented followed by a detailed understanding of the MW drying process in terms of key process parameters. Relationship between various electromagnetics, multiphase transport and solid mechanics were developed as affecting temperature, moisture loss and volume shrinkage during the MW drying process. This was followed by sensitivity analysis of some important transport and solid mechanical properties to understand the need for their precise measurements. A detailed model, such as the one presented in this work, is essential to understand a complex process such as MW drying in order to able to optimize them.

The major conclusions from the work are summarized as follows: (1) The model predicts temperature, moisture and volume changes during the drying process reasonably well. Improvements can be seen in model predictions with the inclusion of solid mechanics into the modeling framework for the first time.

(2) The effective permittivity of the material decreases as moisture is lost during the drying process that manifests itself in lower product temperature during the later portions of the drying process. (3) Microwave power absorption decreases due to a decrease in the effective permittivity and volume of the material during drying. (4) Volume changes is primarily due to moisture loss and there is negligible contribution from gas pressure gradients. (5) Mechanical deformation critically affects dielectric properties; these property values are much lower for a rigid material compared with a deformable material. (6) Power cycling allows for a uniform distribution of temperature, pressure and moisture within the material. (7) Although pressure gradients do not drive the deformation, they are responsible for stress development within the material. (8) Moisture loss and volumetric shrinkage during microwave drying are not particularly sensitive to changes in intrinsic permeability and elastic modulus. However, the stress state within the material highly depends upon elastic modulus values and thus would require accurate measurements.

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Table 1: Input parameters used in the simulations for microwave drying

Parameter	Value	Units	Source
Sample (cube) dimensions, d	$10 \times 10 \times 10$	mm	
Electromagnetics			
Microwave frequency, f	2.45	GHz	
Dielectric constant			
water (1% salt), ϵ'_w	$-0.2833T + 80.67$	-	60
solid, ϵ'_s	4	-	18
gas, ϵ'_g	1	-	
Dielectric loss			
water (1% salt), ϵ''_w	$0.05T + 20$	-	60
solid, ϵ''_s	0.2	-	18
gas, ϵ''_g	0	-	
Transport			
Density			
water, ρ_w	998	kg/m ³	
vapor, ρ_v	Ideal gas	kg/m ³	
air, ρ_a	Ideal gas	kg/m ³	
solid, ρ_s	1528	kg/m ³	66
Specific heat capacity			
water, C_{pw}	$4176.2 - 0.0909(T - 273) + 5.4731 \times 10^{-3}(T - 273)^2$	J/kg·K	68
vapor, C_{pv}	2062	J/kg·K	
air, C_{pa}	1006	J/kg·K	69
solid, C_{ps}	1650	J/kg·K	69
Thermal conductivity			
water, k_w	$0.57109 + 0.0017625T - 6.7306 \times 10^{-6}T^2$	W/m·K	69
vapor, k_v	0.026	W/m·K	69
air, k_a	0.026	W/m·K	69
solid, k_s	0.21	W/m·K	69
Viscosity			
water, μ_w	0.988×10^{-3}	Pa·s	
vapor and air, μ_g	1.8×10^{-5}	Pa·s	
Intrinsic Permeability			
water, $k_{in,w}$	$f(\phi) \times 1 \times 10^{-15}$	m ²	54
air and vapor, $k_{in,g}$	$f(\phi) \times k_{in,w} \left(1 + \frac{0.15k_{in,w}^{-0.37}}{p} \right)$	m ²	56
Relative permeability			
water, $k_{r,w}$	$((S_w - 0.09)/0.91)^3, \quad S_w > 0.09$ $0, \quad S_w < 0.09$		57
air and vapor, $k_{r,g}$	$(1 - 1.1S_w), \quad S_w < 0.91$ $0, \quad S_w > 0.91$		57

Parameter	Value	Units	Source
Capillary diffusivity (water), $D_{w,cap}$	$10^{-8} \exp(-2.8 + 2M)$	m^2/s	48
Equilibrium vapor pressure, $p_{v,eq}$	$p_{sat} \exp(-0.0267M^{-1.656} + 0.0107e^{-1.287M}M^{-1.513} \ln p_{sat})$	Pa	70
Heat transfer coefficient, h_t	10	$\text{W}/\text{m}^2 \cdot \text{K}$	29
Mass transfer coefficient, h_m	0.008	m/s	Chilton-Coulborn analogy
Vapor diffusivity in air, D_{bin}	$\frac{2.13}{p} \left(\frac{T}{273} \right)^{1.8} (S_g \phi)^{3-\phi} / \phi$	m^2/s	49
Ambient pressure, P_{amb}	101,325	Pa	
Latent heat of evaporation, λ_h	2.26×10^6	J/kg	
Evaporation rate constant, K_{evap}	1000	$1/\text{s}$	34
Porosity, ϕ	0.88		
Solid Mechanics			
Poisson's Ratio, ν	0.49		65
Elastic modulus, E	$1.69 \times 10^7 \exp(-0.522M)$	N/m^2	67
Bulk modulus, K	Eqn. 66	N/m^2	
Shear modulus, μ	Eqn. 67	N/m^2	
Initial Conditions			
Pressure, P_0	101,325	Pa	
Water concentration, $c_{w,0}$	835.4	kg/m^3	
Vapor mass fraction, $\omega_{w,0}$	0.026		
Temperature, T_0	25	$^{\circ}\text{C}$	

Figure 1: Schematic showing the oven and sample geometries used for simulations and boundary conditions for coupled electromagnetics – multiphase transport – solid mechanics model. The volume of the potato is 1 cm^3 and that of the microwave chamber is 17016 cm^3 .

Figure 2: Flowchart showing the three-way coupling between electromagnetics, solid mechanics and multi-phase transport for the computational model.

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Figure 6: (a) Oven and material domain, (b) region between the cavity and material where degrees of freedom become zero due to absence of domain when the drying material shrinks, (c) outer domain of the same shape but slightly bigger than the drying material where ALE framework is applied to reduce degrees of freedom, and (d) the inner walls of the outer ALE domain move with the drying material as it shrinks.

(a) Oven mesh

(b) Drying material mesh

Figure 7: Meshed (a) oven geometry and (b) drying material with tetrahedral mesh elements.

(a) Experimental system

(b) Sample with Teflon stick

Figure 8: Schematic of (a) microwave drying experimental system comprising of digital weighing balance for measuring transient moisture loss, fiber optic sensors for measuring point temperature and pressure and infrared camera for mapping surface temperature profiles during microwave drying and (b) sample hung to the weighing balance using a teflon stick.

Figure 9: Computed and experimental temperature history near the center of the material with and without shrinkage effects. Note the wavy nature of the plot showing the effects of microwave cycling.

Figure 10: Experimental and computed surface temperature profiles for the material at different times during the microwave drying process. For clarity purposes, the entire material is also shown even though computations were done on one half of the material owing to symmetry in temperature profiles.

Figure 11: Average moisture content of the material at different times obtained from computations and moisture loss measurements during microwave drying. Also shown is the computed moisture loss history for a material whose volume stays fixed.

Figure 12: Computed and measured changes in surface area of the material as function of moisture at different times. Surface shrinkage curve indicates that shrinkage is equal to the amount of water lost.

(a)

(b)

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Figure 14: Computed changes in dielectric properties as a function of moisture content of the material with and without shrinkage effects. Both the dielectric constant and loss decrease with decreasing moisture. The wavy nature is due to microwave cycling.

Figure 15: Computed gas porosity in the material with and without shrinkage effects. Gas porosity is higher for a rigid material while shrinkage compensates for volume of water lost resulting in very low gas porosity in the deformable material.

Figure 16: Computed power absorption in the rigid and deformable material as function of moisture content. Amount of power absorbed decreases with volume and moisture content. The decrease is more for the rigid material due to a larger decrease in dielectric property as shown in Figure 14. Whereas for the deformable material, the contribution from decrease in volume contributes more towards a decrease in the ability to absorb microwaves.

Figure 17: Computed power absorbed per unit volume in the material at different cross-sections and at different times (8 and 24 cycles) of microwave drying. Profiles are shown for both ON and OFF phases of the cycle.

Figure 18: Computed temperature in the material at different cross-sections and at different times (8 and 24 cycles) of microwave drying. Profiles are shown for both ON and OFF phases of the cycle.

Figure 19: Computed evaporation in the material at different cross-sections and at different times (8 and 24 cycles) of microwave drying. Profiles are shown for both ON and OFF phases of the cycle.

Figure 20: Computed pressure in the material at different cross-sections and at different times (8 and 24 cycles) of microwave drying. Profiles are shown for both ON and OFF phases of the cycle.

Figure 21: Computed liquid water concentration in the material at different cross-sections and at different times (8 and 24 cycles) of microwave drying. Profiles are shown for both ON and OFF phase of the cycle.

Figure 22: Computed effective stress in the material at different cross-sections and at different times (8 and 24 cycles) of microwave drying. Stresses are higher in regions near the surface facing the port. Peak stress values increase with drying time as more moisture is lost.

Figure 23: Computed gas pressure gradients in the material at different cross-sections at different times (8 and 24 cycles) of microwave drying. Pressure gradients are higher in regions near the surface facing the port resulting in higher stress near those regions.

Figure 24: Computed maximum effective stress in the material as a function of moisture content that stays well below the fracture stress of the material. The wavy nature arises due to the effects of microwave cycling. When the microwave is ON, pressures are higher resulting in higher stress levels in the material.

(a) Moisture loss history

(b) Volumetric shrinkage

(c) Moisture loss history

(d) Volumetric shrinkage

Figure 25: Sensitivity of microwave drying process to changes in intrinsic permeability (a and b) and elastic modulus (c and d) using the total moisture loss, and volumetric shrinkage as the variable of interest.

Figure 26: Computed maximum effective stress for different elastic modulus values. Maximum effective stress increases with increase in elastic modulus making the material more prone to fracture (see Figure 24).