Flow and pressure characteristics in rectangular channels with internal cylindrical bodies

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Abstract

This paper develops models for steady-state, fully developed, laminar flow in rectangular channels with internal coaxial solid cylinders. By casting and solving the parallel-flow momentum equation in a dimensionless setting, correlations are derived for the friction factor f and represented as $Re f$ where Re is the Reynolds number. The correlations consider three positions for the internal cylinder. The cylinder may be in the center of the rectangular channel, in the corner of the channel, or resting at the middle of the channel floor. The correlations incorporate channel aspect ratios in the range $0.1 \leq \alpha \leq 1.0$. The cylinder-diameter aspect ratios β range from being vanishingly small to being large enough to touch the channel walls. Although the results are general, the study is motivated by considering the effects of diagnostic probes within small channels of catalytic monoliths.

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1 **1. Introduction**

2 Small-channel devices, such as catalyst monoliths (e.g., Fig. 1), are used
3 in a wide range of technologies. Example applications include exhaust-gas
4 after treatment and catalytic partial oxidation of hydrocarbon fuels. Be-
5 cause individual channel cross-sectional dimensions are typically small (i.e.,
6 order of millimeters), measuring temperature and composition within indi-
7 vidual channels is difficult. A probe-based approach for measuring species
8 composition was first reported by Partridge et al. [1]. Moving the capillary
9 probe along the length of a channel enables the measurement of axially re-
10 solved concentration profiles. The approach has been applied and extended
11 by others and also includes temperature measurements with thermocouple
12 probes [1–13].

13 [Figure 1 about here.]

14 Considering the small dimensions of the channels in a typical monolith,
15 the presence of physical probes may influence the measurement profiles. Ide-
16 ally, “capillary” probes should be sufficiently small and positioned so as to
17 only minimally affect the flow through the channels, and thus the interpreta-
18 tion of the measurement. However, even small variations in channel dimen-
19 sions, and hence the influence of an embedded probe, strongly affect the flow
20 rate and pressure drop within small channels. Results of the present study

21 show that placing the small probe in the corner of a relatively larger chan-
22 nel leads to minimal flow-field perturbations. However, larger or centrally
23 positioned probes can cause major flow restrictions.

24 The modeling approach here is based upon laminar, fully developed,
25 parallel flow within long rectangular channels with an internal solid cylin-
26 der. The two-dimensional momentum equation is formulated and solved in
27 a dimensionless framework. The results predict the dimensionless Reynolds-
28 number–friction-factor product $Re f$ as a function of the channel aspect ra-
29 tio, cylinder diameter, and cylinder placement. Because the model is based
30 upon fully developed flow, the results under-predict the friction factor in
31 developing-flow regions, such as the channel entrance region. Nevertheless,
32 for typically low Reynolds number flow in small channels, the development
33 regions are short. Hettel et al. [9] have developed detailed three-dimensional
34 computational fluid dynamics (CFD) models to evaluate probe effects within
35 catalyst monoliths. These models accurately represent the entry-length de-
36 velopment regions, which the present model neglects by assuming fully de-
37 veloped flow. However, the present model provides easily used correlations
38 for a wide range of channel and probe geometries.

39 Texts on heat exchangers provide friction-factor correlations (i.e., $Re f$)
40 for numerous channel cross-sectional shapes (cf., Shaw and Sekulić [14]).
41 Most basic fluid mechanics texts give correlations for circular and rectangular
42 channels. However, it is rare to find correlations for channels with internal
43 obstructions. Thus, the present results provide new and general correlations.
44 Section 4 provides illustrative examples and Section 5 discusses how to apply
45 the correlations in plug-flow models.

46 The present paper develops the theory and models to evaluate the fluid
 47 mechanical influence of axially aligned cylinders within rectangular flow chan-
 48 nels. The results provide quantitative tools with which to evaluate the extent
 49 of any potential invasive behavior, and thus to assist the engineering design
 50 of a probe system. Although certainly motivated by applications such as
 51 probes in monoliths, the approach and results are more general. Because
 52 of their generality with respect to rectangular-channel dimensions, cylinder
 53 diameter, and cylinder placement, the new correlations are potentially useful
 54 for other, even unforeseen, applications.

55 **2. Model development**

56 The modeling follows a traditional approach of solving a dimensionless
 57 axial momentum equation for laminar, fully developed parallel laminar flow
 58 in a channel [14–16]. The two-dimensional velocity distribution must satisfy
 59 the axial momentum equation as

$$\nabla^2 u = \frac{1}{\mu} \frac{dp}{dz}, \quad (1)$$

60 where u is the velocity, p is the pressure, μ is the dynamic viscosity, and z is
 61 the axial coordinate. Because the flow is fully developed, and assuming con-
 62 stant viscosity, the right-hand side must be a constant. The solution to this
 63 elliptic equation requires boundary conditions. Assuming no-slip conditions,
 64 the velocity on all the solid boundaries (channel walls and probe surface)
 65 vanishes (i.e., $u = 0$).

66 To facilitate transformation to a dimensionless setting, the characteristic
 67 length scale may be represented in terms of a hydraulic diameter or the square

68 root of the cross-sectional area. Solving the partial differential equation yields
 69 Ref as a function of the channel geometry. Figure 2 illustrates a rectangular
 70 channel that measures a high and b wide. A circular probe of diameter d is
 71 placed at the center of the channel. Assume that $a \leq b$ and $d \leq a$. Aspect
 72 ratios for the rectangular channel and the cylinder may be defined as $\alpha = a/b$
 73 and $\beta = d/b$, respectively. These aspect ratios are related and bounded as

$$\beta < \alpha \leq 1, \quad 0 < \beta < \alpha. \quad (2)$$

74 These bounds enforce the constraint that the probe dimensions cannot ex-
 75 ceed those of the rectangular channel and that the cylinder cannot vanish
 76 entirely (i.e., $\beta = 0$). The case of $\beta = 0$ (i.e., rectangular channel with-
 77 out a cylinder) is physically valid, leading to well-known solutions. However,
 78 the present model formulation imposes a no-slip boundary condition at the
 79 cylinder wall, which cannot be imposed in the absence of the cylinder.

80 [Figure 2 about here.]

81 2.1. Characteristic scales

82 Generality can be achieved by casting the problem in dimensionless vari-
 83 ables. Doing so begins with identifying characteristic length and velocity
 84 scales. The most-commonly-used length scale is the hydraulic diameter,
 85 which is defined as

$$\mathcal{L}_{D_h} = \frac{4A_c}{P}, \quad (3)$$

86 where $A_c = ab - \pi d^2/4$ and $P = 2a + 2b + \pi d$ are the channel cross-sectional
 87 area and wetted perimeter, respectively. Represented in terms of the aspect
 88 ratios,

$$\mathcal{L}_{D_h} = b \frac{4\alpha - \pi\beta^2}{2(1 + \alpha) + \pi\beta}. \quad (4)$$

89 Although the hydraulic diameter is widely used, Muzychka and Yovanovich [17]
 90 suggest that the square root of the cross-sectional area is a more appropriate
 91 characteristic length scale for complex channel geometries. That is,

$$\mathcal{L}_{\sqrt{A_c}} = \sqrt{A_c} = \sqrt{ab - \frac{\pi d^2}{4}} = b\sqrt{\alpha - \frac{\pi\beta^2}{4}}. \quad (5)$$

92 The velocity scale is taken to be the mean velocity U . The mean velocity is
 93 related to the net mass-flow rate through the channel as $\dot{m} = \rho U A_c$, where
 94 ρ is the fluid's mass density. Using these characteristic scales, dimensionless
 95 variables may be written as

$$\hat{x} = \frac{x}{\mathcal{L}}, \quad \hat{y} = \frac{y}{\mathcal{L}}, \quad \hat{u} = \frac{u}{U}. \quad (6)$$

96 At this point, the characteristic length scale is represented as $\mathcal{L}$, without
 97 being specific as to using $\mathcal{L} = D_h$ or $\mathcal{L} = \sqrt{A_c}$.

98 2.2. Dimensionless equations

99 Using the characteristic scales, the dimensionless momentum equation
 100 may be written as

$$\nabla^2 \hat{u} = \frac{1}{\mu} \frac{\mathcal{L}^2}{U} \frac{dp}{dz}. \quad (7)$$

101 For the sake of simplifying the nomenclature, the variable Γ is defined as

$$\Gamma \equiv \frac{1}{\mu} \frac{\mathcal{L}^2}{U} \frac{dp}{dz}. \quad (8)$$

102 The mean velocity U can be represented in terms of the local velocity
 103 distribution as

$$U = \frac{1}{A_c} \int_{A_c} u \, dA_c. \quad (9)$$

104 Recognizing that $\hat{A}_c = A_c/\mathcal{L}^2$, transformation to dimensionless variables
 105 yields

$$1 = \frac{1}{\hat{A}_c} \int_{\hat{A}_c} \hat{u} \, d\hat{A}_c. \quad (10)$$

106 Equation 10 represents an integral constraint on the solution of Eq. 7. The
 107 parameter Γ plays the role of an eigenvalue that must be determined such
 108 that Eq. 10 is satisfied. In other words, Eq. 7 has a solution for any value of
 109 Γ . However, the only relevant solution is the one for which Γ is chosen such
 110 that Eq. 10 is satisfied.

111 Because Γ (right hand side of Eq. (7)) is a constant, it is convenient to
 112 define a variable ϕ as

$$\phi \equiv \frac{\hat{u}}{\Gamma}. \quad (11)$$

113 Using this definition, the momentum equation can be rewritten as

$$\nabla^2 \phi = \nabla^2 \left(\frac{\hat{u}}{\Gamma} \right) = 1. \quad (12)$$

114 Equation 12 is a linear elliptic equation that, in principle, can be solved an-
 115 alytically by separation of variables, even on a complex geometry. However,
 116 the analytic solution is difficult to derive, with the result being represented
 117 as a very complicated infinite series. Such an analytic solution does not offer
 118 higher accuracy or other benefits, and a computational solution is much more
 119 easily accomplished. The approach here is to solve the equation computa-
 120 tionally on a finite-element mesh using the PDE toolbox in MATLAB (cf.,
 121 Sect. 2.4). This approach produces highly accurate solutions with very little
 122 computational effort.

123 By using Eqs. 10 and 11, the relevant value for Γ can then be extracted

124 from the solution for ϕ as

$$\Gamma = \frac{\hat{A}_c}{\int_{\hat{A}_c} \phi \, d\hat{A}_c}. \quad (13)$$

125 The full dimensionless area may be evaluated directly as

$$\hat{A}_c = \frac{A_c}{\mathcal{L}^2} = \frac{(ab - \pi d^2/4)}{\mathcal{L}^2} = \frac{b^2 (\alpha - \pi\beta^2/4)}{\mathcal{L}^2}. \quad (14)$$

126 Alternatively, it may be evaluated computationally from the finite-element
127 mesh as

$$\hat{A}_c = \int_{\hat{A}_c} d\hat{A}_c, \quad (15)$$

128 which, by comparing the two values, serves as a check on the accuracy of the
129 numerical quadrature.

130 The dimensionless momentum equation must be solved on the dimen-
131 sionless domain (cf., Fig. 2). Using the definition of the characteristic length
132 scale, the dimensionless domain dimensions are

$$\hat{b} = \frac{b}{\mathcal{L}}, \quad (16)$$

$$\hat{a} = \frac{a}{\mathcal{L}} = \alpha \frac{b}{\mathcal{L}} = \alpha \hat{b}, \quad (17)$$

$$\hat{d} = \frac{d}{\mathcal{L}} = \beta \frac{b}{\mathcal{L}} = \beta \hat{b}. \quad (18)$$

135 Note that the dimensionless domain can be written entirely in terms of the
136 dimensionless aspect ratios α and β (i.e., Eq. 4 or 5). Thus, different channel
137 aspect ratios lead to different dimensionless domains (cf., Fig. 2) for the
138 solution of Eq. 7.

139 2.3. Friction factor

140 As is well known and widely practiced, dimensionless measures of mean
141 wall shear stress within channels and ducts can be represented as the product

142 of Reynolds number and friction factor, Re_f . The general approach to deriv-
 143 ing and using Re_f correlations can be found in most fluid-mechanics books
 144 that deal with internal flows (cf., [14–16]). The objective of the present pa-
 145 per is to derive and evaluate new friction-factor correlations for rectangular
 146 channels with interior cylindrical bodies.

147 The mean shear stress τ_m associated with fluid-mechanical drag at the
 148 channel walls and the probe surface can be written in integral form as

$$\tau_m = \frac{1}{P} \int_P \mu \nabla u \cdot \mathbf{n} ds, \quad (19)$$

149 where s is the distance around the perimeter, $P = 2a + 2b + \pi d$ is the
 150 wetted perimeter, and $\mathbf{n}$ is an outward-pointing normal unit vector at the
 151 solid surfaces around the perimeter. The mean shear stress can be written
 152 in dimensionless form using a friction factor f that is defined as

$$f \equiv \frac{\tau_m}{\frac{1}{2}\rho U^2}. \quad (20)$$

153 It follows directly that

$$f = \frac{2}{\rho U^2} \left(\frac{1}{P} \int_P \mu \nabla u \cdot \mathbf{n} ds \right). \quad (21)$$

154 The right-hand side of Eq. 21 can be transformed to dimensionless variables,
 155 thus enabling the direct use of the dimensionless solution to the momentum
 156 equation (Eq. 7). Considering the definition of the Reynolds number as

$$Re = \frac{\rho U \mathcal{L}}{\mu}, \quad (22)$$

157 an expression for Re_f follows as

$$f = \frac{2}{\rho U^2} \frac{\mu U}{\mathcal{L}} \left(\frac{1}{\hat{P}} \int_{\hat{P}} \nabla \hat{u} \cdot \mathbf{n} d\hat{s} \right), \quad (23)$$

158

$$f = \frac{2}{\text{Re}} \left(\frac{1}{\hat{P}} \int_{\hat{P}} \nabla \hat{u} \cdot \mathbf{n} d\hat{s} \right), \quad (24)$$

159

$$\text{Ref} = \frac{2}{\hat{P}} \int_{\hat{P}} \nabla \hat{u} \cdot \mathbf{n} d\hat{s}. \quad (25)$$

160 The dimensionless perimeter may be written as

$$\hat{P} = \frac{P}{\mathcal{L}} = \frac{b}{\mathcal{L}} (2(1 + \alpha) + \pi\beta). \quad (26)$$

161 For any solution of the momentum equation (i.e., $\hat{u}(\hat{x}, \hat{y})$), it is evident from
 162 Eq. 25 that Ref is a constant. Although evaluating the perimeter integral
 163 (Eq. 25) can be accomplished computationally, it can be cumbersome for
 164 the complex geometry. Thus, as explained in the following paragraph, an
 165 easier-to-implement alternative can be developed.

166 A force balance on an axial differential section of the fluid within the
 167 rectangular channel can be written as

$$A_c \frac{dp}{dz} = P\tau_m = P \frac{1}{P} \int_P \mu \nabla u \cdot \mathbf{n} ds. \quad (27)$$

168 In other words, the pressure gradient acting on the fluid cross section is bal-
 169 anced by the shear drag associated with the channel walls and the cylindrical
 170 probe surface. Transforming to dimensionless variables yields

$$\frac{\mathcal{L}^2}{\mu U} \frac{dp}{dz} = \frac{1}{\hat{A}_c} \int_{\hat{P}} \nabla \hat{u} \cdot \mathbf{n} d\hat{s}. \quad (28)$$

171 Equation 25 can be substituted for the perimeter integral, yielding

$$\frac{\mathcal{L}^2}{\mu U} \frac{dp}{dz} = \frac{1}{2} \frac{\hat{P}}{\hat{A}_c} \text{Ref}. \quad (29)$$

172 Using the definition of the eigenvalue Γ (Eq. 8),

$$\text{Ref} = \frac{\mathcal{L}^2}{\mu U} \frac{dp}{dz} \frac{2\hat{A}_c}{\hat{P}} = \frac{2\hat{A}_c}{\hat{P}} \Gamma. \quad (30)$$

173 The dimensionless cross-sectional area and perimeter are stated in Eqs. 14
 174 and 26, respectively. Thus, once a geometry is specified and the dimension-
 175 less momentum equation is solved, establishing the eigenvalue Γ , the value
 176 of the constant Ref follows directly. Using the hydraulic diameter as the
 177 characteristic length scale (Eq. 4) and using Eqs. 14 and 26 for $\hat{A}_c$ and $\hat{P}$,
 178 respectively, it follows that $\hat{A}_c/\hat{P} = 1/4$. Thus, Eq. 30 simplifies as

$$Ref_{D_h} = \frac{\Gamma}{2}. \quad (31)$$

179 The Reynolds number in the analysis, and hence in Eq. 30, is based on the
 180 characteristic length scale $\mathcal{L}$. However, as discussed earlier, the characteristic
 181 length may be defined alternatively as the hydraulic diameter $\mathcal{L}_{D_h}$ or the
 182 square root of the cross-sectional area $\mathcal{L}_{\sqrt{A_c}}$. Depending on the length-scale
 183 definition, the dimensionless computational domain changes. Furthermore,
 184 the interpretation of Ref depends on the length-scale definition (i.e., Ref_{D_h}
 185 or $Ref_{\sqrt{A_c}}$).

186 [Figure 3 about here.]

187 2.4. Computational Procedure

188 Equation 12 is a linear equation that is solved computationally within
 189 the Matlab PDE toolbox using a finite-element method with adaptive mesh
 190 refinement. High accuracy is assured by mesh refinement to the point that
 191 the solutions are independent of mesh resolution. The solutions use between
 192 10,000 and 100,000 finite elements. Each solution requires only a few seconds
 193 on a typical personal computer.

194 The equations are solved numerous times for different probe placements
 195 and aspect ratios. Figure 3 shows three representative solutions, each with

196 a different probe placement. In all these cases, the aspect ratios are $\alpha = 1$
 197 and $\beta = 0.2$. The contours show scaled dimensionless velocity maps. It is
 198 evident that the velocities are much lower (hence more shear drag) when the
 199 probe is placed in the center of the channel. Consequently, the mean velocity
 200 (Eq. 9) is lower when the cylinder is more centrally located.

201 [Figure 4 about here.]

202 [Figure 5 about here.]

203 **3. Ref correlations**

204 Figure 4 shows Ref_{D_h} correlations as functions of the cylinder aspect ratio
 205 β ($\beta > 0.001$) for a range of rectangular-channel aspect ratios α . In all cases,
 206 the cylinder is at the center of the rectangular channel. At very small values
 207 of β the Ref_{D_h} correlation becomes very steep.

208 When $\beta = 0$, the central cylinder disappears. Thus, the no-slip (zero-
 209 velocity) boundary condition at the cylinder surface no longer applies. Con-
 210 sequently, the problem becomes singular if a no-slip boundary condition is
 211 applied at a surface that does not exist. As discussed subsequently, the prob-
 212 lem can be solved without an interior cylindrical body (e.g., probe), yielding,
 213 for example, the well-known result of $Ref_{D_h} = 14.22$ when $\alpha = 1$.

214 The correlations shown in Fig. 4 have some possibly unanticipated char-
 215 acteristics, which result from the definitions of the length scales in the dimen-
 216 sionless analysis. The Ref_{D_h} functions all reach a maximum value at some β ,
 217 and then decrease at high β . As β increases, the cylinder occupies a larger
 218 fraction of the channel. Thus, as the flow area decreases the net shear stress

219 and pressure drop must increase for particular mass flow rates. However,
 220 it is interesting that Re_{D_h} decreases as β increases toward its maximum
 221 value (i.e., where cylinder touches a channel wall in dimensionless space). At
 222 $\beta \approx 0.055$, the functions all cross, with Re_{D_h} being essentially independent
 223 of α . The functional forms of Re_{D_h} are the result of the hydraulic-diameter
 224 definition and the dimensionless analysis. Despite the fact that the dimensionless
 225 Re_{D_h} functions have local maxima (Figs. 4-7), the physical behaviors
 226 do not have such local maxima. The examples discussed in Sect. 4 show the
 227 expected behaviors. That is, for example, as the cylinder becomes larger the
 228 pressure drop increases monotonically.

229 Figure 5 shows Re_{D_h} correlations as functions of the cylinder aspect ratio
 230 β for a range of rectangular-channel aspect ratios α . In all cases, the cylinder
 231 is in the corner of the rectangular channel. Compared to Fig. 4, the Re_{D_h}
 232 functions are qualitatively very different. Placing the cylinder in the corner
 233 leads to Re_{D_h} mostly decreasing monotonically as a function of β . However,
 234 there are weak local minima for $\alpha > 0.8$ and large β . For most of the β
 235 range, the Re_{D_h} values are lower for the corner cases than they are for the
 236 centrally placed cylinder.

237 Figure 6 shows Re_{D_h} correlations as functions of the cylinder aspect
 238 ratio β for a range of rectangular-channel aspect ratios α . In all cases, the
 239 cylinder is on the middle of the floor of the rectangular channel. Again, the
 240 correlations are qualitatively different from those shown in Figs. 4 and 5.

241 Although Figs. 4-6 show correlations for wide ranges of the rectangular-
 242 channel aspect ratios, practical catalyst monoliths almost always use square
 243 channels (i.e., $\alpha = 1$). Figure 7 shows correlations for $\alpha = 1$, but with

244 different cylinder diameters β and placements.

245 At $\beta = 0$ (i.e., open channel without a cylinder), the results show $Ref_{D_h} \approx$
246 14.22, which is the well-known result for fully developed flow in a square
247 channel [14, 15]. At large β , where the cylinder nearly touches the channel
248 walls, the results for all three cylinder positions approach $Ref_{D_h} \approx 8$. From
249 the viewpoint of monolith-sampling technology, this result (i.e., the cylinder
250 filling the channel as much as possible) has relatively little practical utility.
251 However, there may well be other circumstances in which a cylinder that fills
252 a rectangular channel is relevant. Table 1 lists polynomial fitting coefficients
253 that replicate the results ($\alpha = 1$) that are shown graphically in Fig. 7.

254 As β approaches zero (i.e., the cylindrical probe vanishes), the asymptotic
255 behavior of Ref is qualitatively different in the case of the centrally placed
256 cylinder than it is when the cylinder is adjacent to a channel wall. A no-slip
257 boundary condition is applied to the cylinder wall, which serves to retard
258 the flow in the vicinity of the cylinder. For centrally positioned cylinders,
259 even with very small cylinder diameters (i.e., small β), the no-slip condition
260 retards the flow in the central regions of the channel. At $\beta = 0$ (i.e., without
261 a cylinder) the no-slip boundary condition at the cylinder wall is no longer
262 applicable and the problem must be solved for the channel alone without a
263 cylinder. In other words, the fundamental problem formulation and boundary
264 conditions must change at $\beta = 0$. However, when the cylinder is adjacent
265 to the channel wall (bottom and corner cases), the situation is different. As
266 the cylinder diameter becomes smaller and smaller, the no-slip boundary
267 condition still applies. However, in this case the no-slip boundary condition
268 at the cylinder surface becomes indistinguishable from the no-slip boundary

condition at the cylinder wall. Thus, the Ref solutions smoothly approach the open-channel solutions.

Although the shapes of the Ref correlations shown in Fig. 7 are different, the reasons for the common end points are evident. When the cylinder vanishes (i.e., $\beta = 0$), all the solutions must converge to the open-channel solution (i.e., $Ref=14.22$). When the cylinder becomes as large as possible (i.e., touching the channel walls), again all the Ref solutions must be the same. Regardless of whether the cylinder begins in the center, at the corner, or on the mid bottom, for $\alpha = 1$ the geometries are identical when $\beta = 1$. The only open flow spaces are the approximately triangular regions in the channel corners. The correlation $Ref_{D_h} \approx 8$ of this particular geometric configuration is a new result of the present paper.

[Figure 6 about here.]

[Figure 7 about here.]

[Figure 8 about here.]

3.1. Prior Ref correlations

Figure 8 illustrates Ref correlations for the well-known cases of flow in open rectangular channels and cylindrical-annulus ducts [14, 15]. In both cases the Reynolds number is based on hydraulic diameter. In open rectangular channels the correlation decreases monotonically from $Ref = 24$ to approximately $Ref \approx 14.2$. In the annular case, the correlation function increases monotonically from $Ref = 16$ (circular duct without a center cylinder) to $Ref = 24$ (asymptotic result for parallel plates). The inset equations

show analytic expressions for the correlations. The relatively simple expression for the open rectangular channel was taken from Kee, et al. [18]; a more-accurate five-parameter polynomial fit can be found in Shah and Sekulić [14].

Intuitively, one might expect the functional behaviors seen in Figs. 4-7 to be similar to those seen in Fig. 8. However, with two independent geometric parameters (α and β), the correlations are inherently more complex than those of the well-known simpler geometric configurations with only one geometric parameter (e.g., Fig. 8)

3.2. Length scale based on $\sqrt{A_c}$

Dimensionless analysis necessarily depends on characteristic scales. The hydraulic diameter (Eq. 3) is by far the most commonly used length scale for analyzing flow in channels and ducts. However, there is no requirement that any particular scale be used. Alternative characteristic scales may lead to better or more-easily-used correlations. Muzychka and Yovanovich [17] suggest advantages associated with using the square root of the cross-sectional area as the characteristic length scale. Thus, it is reasonable to investigate the possibility that $\mathcal{L} = \sqrt{A_c}$ might produce simpler or more intuitive function forms for the Ref correlations.

Using $\mathcal{L} = \sqrt{A_c}$, Fig. 9 shows $Ref_{\sqrt{A_c}}$ as functions of β for the square channel cross sections. The functional forms of $Ref_{\sqrt{A_c}}$ are significantly different from function forms for Ref_{D_h} based on the hydraulic diameter as shown in Fig. 7. However, both correlations yield relatively complicated nonlinear functional forms for Ref . Thus, with no apparent advantage for using $\mathcal{L} = \sqrt{A_c}$, the paper presents the results in terms of the hydraulic diameter being the characteristic length scale. Of course, the physical results

(e.g., pressure drops) are independent of the characteristic length scales that are used in formulating the dimensionless problem.

[Table 1 about here.]

[Figure 9 about here.]

3.3. Entry region

The foregoing analysis is based upon fully developed flow. However, in fact, as uniform flow enters a channel, boundary layers form on the channel walls. The flow becomes fully developed only after some entry-length region. In the entry region, the friction factor is higher than it is once the flow becomes fully developed. Thus, it is important to evaluate the validity associated with assuming fully developed flow.

Muzychka and Yovanovich [17] develop a general analysis to predict the variation of Re_f in the entry regions of a wide range of noncircular ducts. The general expression is written as

$$Re_{f\sqrt{A_c}} = \sqrt{\left(\frac{3.44}{\sqrt{L^+}}\right)^2 + \left(\frac{12}{\sqrt{\varepsilon}(1+\varepsilon)\left[1 - \frac{192\varepsilon}{\pi^5} \tanh\left(\frac{\pi}{2\varepsilon}\right)\right]}\right)^2}. \quad (32)$$

where the dimensionless length from the channel entrance is

$$L^+ = \frac{L}{\sqrt{A_c}} \frac{1}{Re_{\sqrt{A_c}}}, \quad (33)$$

and ε is a general representation of the channel aspect ratio. However, ε as defined by Muzychka and Yovanovich is different from the aspect ratios α and β . For channels such as the ones considered here, there is no precise definition of ε . Nevertheless, Muzychka and Yovanovich suggest an approximation as

$$\varepsilon = \frac{1 - \sqrt{A_i/A_o}}{\pi \left(1 + \sqrt{A_i/A_o}\right)}, \quad (34)$$

where A_i and A_o are the areas of inner and outer “polygons” that bound the flow area. Here, $A_i = \pi d^2/4$ and $A_o = a \times b$. Substituting the definitions for α and β , the area ratio in Eq. 34 can be rewritten as

$$\sqrt{\frac{A_i}{A_o}} = \sqrt{\frac{\pi \beta^2}{4 \alpha}}. \quad (35)$$

Based on Eq. 32, Fig. 10 shows predicted Re_f as a function of L^+ for two different aspect ratios ε . The asymptotic fully developed value of Re_f is reached at $L^+ \approx 0.1$. Thus, the actual flow-development length depends on the Reynolds number, with lower Reynolds numbers leading to more-rapid approach to the fully developed flow.

Because of relatively small characteristic diameters and low flow rates, small channel devices tend to operate at low Reynolds numbers. For example, at $Re = 10$, the entry-length region is only about one channel diameter. At $Re = 100$, the entry-length region is approximately ten channel diameters. Thus, although some caution needs to be exercised, the fully developed solutions are expected to have practical utility, even in relatively short channels and ducts.

[Figure 10 about here.]

Consider for example, the partial-oxidation catalyst monolith reported by Diehm and Deutschmann [10]. The monolith had 600 square cells per square inch, leading to individual channel dimensions of approximately 1 mm. The inlet flow velocities were approximately 1 m s^{-1} . Assuming properties of air at 900 K, the viscosity is $\mu \approx 4 \times 10^{-5} \text{ Pa s}$ and the mass density is $\rho = 0.4 \text{ kg m}^{-3}$. These conditions lead to a Reynolds number of $Re \approx 10$.

Thus, even for relatively high flow rates associated with partial-oxidation catalysts, the hydrodynamic entry length is only about one channel diameter. The monolith reported by Diehm and Deutschmann was 11 mm long. Thus, approximately 90% of the channel was in the fully developed flow regime. The analysis presented in this paper is generally consistent with the relatively more complicated three-dimensional computational fluid dynamics simulations by Hettel et al. [9].

4. Illustrative examples

Illustrative examples serve to demonstrate the use of the correlations. Continue to consider the partial-oxidation catalyst monolith reported by Diehm and Deutschmann [10]. The monolith had 600 square cells per square inch, resulting in individual channel dimensions of approximately $a = b = 1$ mm ($\alpha = 1$). For an open square channel, $\text{Re}f_{D_h} = 14.2$ [15]. Using the reported mean flow velocities of approximately $U = 1 \text{ m s}^{-1}$ and assuming properties of air at 900 K ($\mu \approx 4 \times 10^{-5} \text{ Pa s}$ and $\rho = 0.4 \text{ kg m}^{-3}$), the Reynolds number for a single monolith channel is

$$\text{Re} = \frac{\rho U D_h}{\mu} = 10, \quad (36)$$

where the hydraulic diameter is $D_h = 4A_c/P = 1 \text{ mm}$. The friction factor easily follows as

$$f = \frac{\text{Re}f}{\text{Re}} = 1.42. \quad (37)$$

The mean shear stress can be evaluated (Eq. 21) as

$$\tau_m = \frac{1}{2}\rho U^2 f = 0.284 \text{ Pa}. \quad (38)$$

377 The force balance on an axial section of the fluid within the channel can be
 378 written as

$$A_c \frac{dp}{dz} = P\tau_m, \quad (39)$$

379 where $A_c = 1 \text{ mm}^2$ and $P = 4 \text{ mm}$ are the channel cross-sectional area and
 380 wetted perimeter, respectively. For a channel length of $L = 20 \text{ mm}$, the
 381 predicted pressure drop is 23 Pa . It is reasonable to assume that each of the
 382 unobstructed channels in the monolith experience the same pressure drop.
 383 Thus the pressure difference across the monolith is $\Delta p = 23 \text{ Pa}$.

384 Assume now that a cylindrical probe is placed within a single channel
 385 of the monolith. The pressure drop is fixed, but shear stress will increase
 386 and the velocity will decrease. Diehm and Deutschmann [10] used a $170 \text{ }\mu\text{m}$
 387 diameter capillary probe ($\beta = 0.17$). Using either Fig. 4 or Table 1, with
 388 $\alpha = 1$, $\beta = 0.17$ and a centered probe, $Re_f \approx 20.7$. Using a rearrangement of
 389 Eq. 30, the mean velocity for the obstructed channel is reduced from $U = 1$
 390 m s^{-1} to

$$U = 2 \frac{\Delta p}{L} \frac{A_c}{P} \frac{D_h}{\mu Re_f} = 0.5 \text{ m s}^{-1}, \quad (40)$$

391 where the definitions of P , A_c , and D_h now include the probe. The velocity
 392 in the obstructed channel is half of that of the other channels.

393 Consider now the case where the probe lies in the corner of the channel,
 394 not in the center. Using Fig. 4 or Table 1, $Re_f = 10.8$. In this case, the
 395 velocity decrement is very small at $U = 0.98 \text{ m s}^{-1}$. Thus, it is clear that in
 396 this application a corner-placed probe produces only a minimal perturbation
 397 of the flow, and hence pressure drop. Placing the probe on the bottom middle
 398 of the channel floor also produces only a small flow perturbation. Of course,
 399 as a practical matter, probe placement in any particular experiment may

400 need to consider other factors beyond flow perturbation and shear drag.

401 Diehm and Deutschmann [10] also used a 630 μm diameter sheathed
402 thermocouple, leading to $\beta = 0.63$. In this case, the center-position probe
403 leads to $\text{Ref} = 21.8$ and a mean velocity of $U = 0.138 \text{ m s}^{-1}$. The corner-
404 position probe leads to $\text{Ref} = 8.35$ and a higher mean velocity of $U = 0.36$
405 m s^{-1} .

406 Figure 11 summarizes the effect of probe diameter and position on the
407 mean velocity for this example. Clearly, the corner position is the least intru-
408 sive, followed by the middle-bottom position, and then the center position.
409 Alternatively, for a specified mass flow rate through the channel, the corner
410 probe requires a lower pressure drop than is required for the center of middle
411 probe positions. As the probe diameter increases, the velocity for all posi-
412 tions decreases. For a square channel ($\alpha = 1$), as the probe entirely fills the
413 channel (i.e., $\beta = 1$), all the probe “positions” approach being the same.

414 Finally, consider the center-cylinder correlations shown in Fig. 4, where
415 all the correlating functions cross at $\beta \approx 0.055$. Figure 12 shows two channels
416 with different aspect ratios, but with the same centered cylinder diameter.
417 Both have a horizontal dimension of $b = 2 \text{ mm}$ and a cylinder diameter of
418 $d = 0.11 \text{ mm}$. The upper channel has aspect ratio $\alpha = 1$ and the lower
419 channel has aspect ratio $\alpha = 0.1$. Thus, referring to Fig. 4, it is evident that
420 these two channels are both at the crossing point where $\text{Ref} \approx 19$. Although
421 the friction factor correlations are identical, it is clear that the flow behaviors
422 for these channels must be quite different. Assume further that both channels
423 are 50 mm long and that a 50 Pa pressure difference is imposed between the
424 channel entrances and exits. Continuing to use the properties of air at 900

425 K, Eq. 40 predicts mean velocities of $U = 2.4 \text{ m s}^{-1}$ for the larger $\alpha = 1$
 426 channel and $U = 0.07 \text{ m s}^{-1}$ for the smaller $\alpha = 0.1$ channel.

427 [Figure 11 about here.]

428 [Figure 12 about here.]

429 5. Plug-flow models

430 An important application of friction-factor correlations is to evaluate lo-
 431 cal wall shear stress in low-dimension plug-flow models. The friction-factor
 432 correlations themselves are derived with two-dimensional models, assuming
 433 indefinitely long channels and correspondingly long central cylinders. In
 434 practice, finite-length probes may be inserted into channels. In other words,
 435 some portion of the channel may contain a probe cylinder and other portions
 436 may not. Moreover, in models that also include thermal and chemical behav-
 437 ior, the mass density ρ and viscosity μ may vary axially along the channel
 438 length.

439 Along the length of a channel, a steady-state plug-flow momentum equa-
 440 tion can be represented as

$$\frac{d(\rho U^2)}{dz} = -\frac{dp}{dz} - \tau_m \frac{P}{A_c}, \quad (41)$$

441 where ρ is the mass density, U is the mean velocity, p is the pressure, τ_m is the
 442 local mean wall shear stress, P is the wetted perimeter, and A_c is the cross-
 443 sectional flow area. The Ref correlations may be used to evaluate the local
 444 wall shear stress. The local Reynolds number (Eq. 22) can be evaluated
 445 in terms of the local mean velocity U (i.e., the dependent variable of the

momentum equation). Thus, knowing the local friction-factor correlation,
the local friction-factor itself is evaluated as

$$f = \frac{Ref}{Re}, \quad (42)$$

where Ref is a constant evaluated from the correlations. As the geometry
varies along the length of the channel (e.g., with or without a probe), the
 Ref correlation varies. Moreover, if the viscosity or density vary axially,
then the local Reynolds number varies axially. The local mean shear stress
is evaluated from the local friction factor using Eq. 21. The plug-flow model
does not account for “entry length” transitions between abrupt changes in
the local channel configuration. However, for low Reynolds number flows the
transitions are rapid (cf., Fig. 10).

6. Summary and conclusions

Motivated by a need to understand how in-situ diagnostic probes affect
flow through small channels, the present analysis develops general friction-
factor correlations for steady-state, fully developed, laminar flow in rectan-
gular channels with interior cylindrical bodies. The analysis incorporates the
influence of alternative flow conditions and geometrical configurations (i.e.,
channel aspect ratio and ratio of probe diameter and channel width). The
correlations are represented in a traditional way as the product of Reynolds
number and friction factor Ref , with the characteristic length scale being
the hydraulic diameter. The study considers three placements of the interior
cylinder. A centrally placed cylinder causes the greatest flow restriction, and
is thus most invasive as a diagnostics probe. Locating the cylinder in the

corner of the rectangular channel causes the least flow disturbance. Placing the probe at the center of the channel floor causes an intermediate flow restriction, but is still much less perturbative than a centrally placed probe. The correlations enable direct quantification of a probe that extends through the full length of a channel. In practice, however, probes may be only partially inserted. Using a plug-flow model, the correlations can also be used to evaluate the influence of partially inserted probes and flows where the fluid properties vary along the channel length.

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551 List of Figures

552	1	Catalytic monolith with a capillary probe within a single chan-	
553		nel.	30
554	2	Channel and probe geometry and layout. The rectangular	
555		channel is a high and b wide. The probe has a diameter d . All	
556		physical dimensions are written in dimensionless form, nor-	
557		malized by the characteristic length $\mathcal{L}$. For example, $\hat{a} = a/\mathcal{L}$	31
558	3	Dimensionless velocity contour maps for three cylinder place-	
559		ments. In all cases, the rectangle aspect ratio is $\alpha = 1$ and the	
560		cylinder aspect ratio is $\beta = 0.2$. The dimensionless-velocity	
561		scale bar is the same for each solution.	32
562	4	Reynolds-number–friction-factor product Ref_{D_h} as a function	
563		of the cylinder-diameter aspect ratio β for ten values of the	
564		rectangular channel aspect ratio α . The cylinder is placed	
565		in the middle of the rectangular channel. The shaded circles	
566		indicate how the cylinder diameter changes as β changes.	33
567	5	Reynolds-number–friction-factor product Ref_{D_h} as a function	
568		of the cylinder-diameter aspect ratio β for ten values of the	
569		rectangular channel aspect ratio α . The cylinder is placed	
570		in the corner of the rectangular channel. The shaded circles	
571		indicate how the cylinder diameter changes as β changes.	34
572	6	Reynolds-number–friction-factor product Ref_{D_h} as a function	
573		of the cylinder-diameter aspect ratio β for ten values of the	
574		rectangular channel aspect ratio α . The cylinder is placed on	
575		the middle of the floor of the rectangular channel. The shaded	
576		circles indicate how the cylinder diameter changes as β changes.	35
577	7	Reynolds-number–friction-factor product Ref_{D_h} as a function	
578		of the cylinder-diameter aspect ratio β for rectangular-channel	
579		aspect ratio $\alpha = 1.0$. The cylinder is placed either in the center	
580		of the channel, in the corner of the channel, or at the middle	
581		of the floor of the rectangular channel.	36
582	8	Well-known Ref correlations for fully developed flow in a)	
583		open rectangular channels and b) cylindrical annuli. In both	
584		cases, the Reynolds number is based on hydraulic diameter.	37

585	9	Reynolds-number-friction-factor product $Re f_{\sqrt{A_c}}$ as a function	
586		of the cylinder-diameter aspect ratio β for rectangular-channel	
587		aspect ratio $\alpha = 1.0$. The cylinder is placed either in the center	
588		of the channel, in the corner of the channel, or at the middle	
589		of the floor of the rectangular channel.	38
590	10	$Re f$ as a function of L^+ as predicted from Eq. 32 for two	
591		different aspect ratios.	39
592	11	Mean inlet velocity as a function of cylinder aspect ratio β	
593		for three probe positions. The pressure drop across a 20 mm	
594		channel length is fixed at $\Delta p = 23$ Pa. The square channel is	
595		1 mm on a side ($\alpha = 1$), and fluid properties of air at 900 °C	
596		are assumed.	40
597	12	Two rectangular channels with a centered cylinder, both hav-	
598		ing $\beta \approx 0.055$. The upper channel has aspect ratio $\alpha = 1$ and	
599		the lower channel has aspect ratio $\alpha = 0.1$	41

Figure 1: Catalytic monolith with a capillary probe within a single channel.

Figure 2: Channel and probe geometry and layout. The rectangular channel is a high and b wide. The probe has a diameter d . All physical dimensions are written in dimensionless form, normalized by the characteristic length $\mathcal{L}$. For example, $\hat{a} = a/\mathcal{L}$.

Figure 3: Dimensionless velocity contour maps for three cylinder placements. In all cases, the rectangle aspect ratio is $\alpha = 1$ and the cylinder aspect ratio is $\beta = 0.2$. The dimensionless-velocity scale bar is the same for each solution.

Figure 4: Reynolds-number-friction-factor product $Re f_{D_h}$ as a function of the cylinder-diameter aspect ratio β for ten values of the rectangular channel aspect ratio α . The cylinder is placed in the middle of the rectangular channel. The shaded circles indicate how the cylinder diameter changes as β changes.

Figure 5: Reynolds-number-friction-factor product $Re f_{D_h}$ as a function of the cylinder-diameter aspect ratio β for ten values of the rectangular channel aspect ratio α . The cylinder is placed in the corner of the rectangular channel. The shaded circles indicate how the cylinder diameter changes as β changes.

Figure 6: Reynolds-number-friction-factor product $Re f_{D_h}$ as a function of the cylinder-diameter aspect ratio β for ten values of the rectangular channel aspect ratio α . The cylinder is placed on the middle of the floor of the rectangular channel. The shaded circles indicate how the cylinder diameter changes as β changes.

Figure 7: Reynolds-number-friction-factor product Ref_{D_h} as a function of the cylinder-diameter aspect ratio β for rectangular-channel aspect ratio $\alpha = 1.0$. The cylinder is placed either in the center of the channel, in the corner of the channel, or at the middle of the floor of the rectangular channel.

Figure 8: Well-known Re_f correlations for fully developed flow in a) open rectangular channels and b) cylindrical annuli. In both cases, the Reynolds number is based on hydraulic diameter.

Figure 9: Reynolds-number-friction-factor product $Re f_{\sqrt{A_c}}$ as a function of the cylinder-diameter aspect ratio β for rectangular-channel aspect ratio $\alpha = 1.0$. The cylinder is placed either in the center of the channel, in the corner of the channel, or at the middle of the floor of the rectangular channel.

Figure 10: $Re f$ as a function of L^+ as predicted from Eq. 32 for two different aspect ratios.

Figure 11: Mean inlet velocity as a function of cylinder aspect ratio β for three probe positions. The pressure drop across a 20 mm channel length is fixed at $\Delta p = 23$ Pa. The square channel is 1 mm on a side ($\alpha = 1$), and fluid properties of air at 900 °C are assumed.

Figure 12: Two rectangular channels with a centered cylinder, both having $\beta \approx 0.055$. The upper channel has aspect ratio $\alpha = 1$ and the lower channel has aspect ratio $\alpha = 0.1$.

600 **List of Tables**

601	1	Coefficients for 7th order polynomial fits for $\alpha = 1$, $0.01 <$
602		$\beta < 1.0$, and three cylinder positions.
603		$\text{Ref}_{D_h}(\beta) = a_0 + a_1 \cdot \beta + a_2 \cdot \beta^2 + a_3 \cdot \beta^3 + a_4 \cdot \beta^4 + a_5 \cdot \beta^5 +$
604		$a_6 \cdot \beta^6 + a_7 \cdot \beta^7$
605		43

Table 1: Coefficients for 7th order polynomial fits for $\alpha = 1$, $0.01 < \beta < 1.0$, and three cylinder positions.

$$\text{Ref}_{D_h}(\beta) = a_0 + a_1 \cdot \beta + a_2 \cdot \beta^2 + a_3 \cdot \beta^3 + a_4 \cdot \beta^4 + a_5 \cdot \beta^5 + a_6 \cdot \beta^6 + a_7 \cdot \beta^7$$

	center	bottom	corner
a_0	17.41	14.27	14.36
a_1	44.3	-24.58	-29.02
a_2	-274.3	139.8	86.35
a_3	1095	-565.4	-397.1
a_4	-2548	1502	1384
a_5	3332	-2346	-2527
a_6	-2267	1955	2266
a_7	608.9	-668.5	-789.8