Distributed Average Tracking for Double-integrator Multi-agent Systems with Reduced Requirement on Velocity Measurements

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Abstract

This paper addresses distributed average tracking for a group of physical double-integrator agents under an undirected graph with reduced requirement on velocity measurements. The idea is that multiple agents track the average of multiple time-varying input signals, each of which is available to only one agent, under local interaction with neighbors. We consider two cases. First, a distributed algorithm and filter are proposed, where each agent needs its own and neighbors' filter outputs obtained through communication besides its local relative positions and its input signal, input velocity and input acceleration. Here, the requirement for either absolute or relative velocity measurements is removed. The algorithm is robust to initialization errors and can deal with a wide class of input signals with bounded deviations in input signals, input velocities, and input accelerations. Second, a distributed algorithm and filter are proposed to remove the requirement for communication. Here, each agent needs to measure the relative positions between itself and its neighbors and its own velocity. However, the requirement for relative velocity measurements between the agent and its neighbors is removed. The algorithm is robust to initialization errors and can deal with the case, where the input signals, input velocities and input accelerations are all bounded.

Key words: Distributed Average Tracking, Cooperative Control, Multi-agent Systems.

1 INTRODUCTION

This paper studies the following distributed average tracking problem: given a group of agents and one time-varying input signal per each agent, design a control law for the agents based on local information such that all the agents will finally track the average of these input signals. The problem has found applications in distributed sensor fusion [1], feature-based map merging [2], and distributed Kalman filtering [3], where the scheme has been mainly used as an estimator. However, there are some applications such as region following formation control [4], coordinated path planning [5] or distributed convex optimization [6] that require the agents'

physical states instead of estimator states to converge to a time-varying network quantity, where each agent only has a local and incomplete copy of that quantity. Compared with the consensus and distributed tracking problems, distributed average tracking poses more theoretical challenges, since the tracking objective is time-varying and is not available to any agent.

In the literature, linear distributed algorithms have been employed for special kinds of time-varying input signals. Ref. [7] uses frequency domain analysis to study consensus on the average of multiple input signals with steady-state values. In [8], a proportional algorithm and a proportional-integral algorithm are proposed to achieve the distributed average tracking with bounded tracking error, where accurate estimator initialization is relaxed in the proportional-integral algorithm. In [9], the internal model principle is employed to extend the proportional-integral algorithm to a special group of time-varying input signals with a common denominator in their Laplace transforms, where the denominator also needs to be used in the estimator design. In [10], the authors propose 1st-order-input and 2nd-order-input consensus algorithms to allow the agents to track the average of their dynamic input signals with a pre-specified rate, where the

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interaction is described by a strongly connected and weight-balanced directed graph. Ref. [11] addresses discrete-time distributed average tracking of time-varying input signals whose n th order difference is bounded with a bounded error. However, linear algorithms cannot ensure distributed average tracking for general input signals. Therefore, some researchers employ nonlinear tracking algorithms. In [12], a class of nonlinear algorithms is proposed for input signals with bounded deviations, where the tracking error is proved to be bounded. A nonsmooth algorithm is proposed in [13], which is able to track time-varying input signals with bounded derivatives.

All the above references primarily study the distributed average tracking problem from a distributed estimation perspective, where the agents implement local estimators through communication with neighbors freely without the need for obeying certain physical agent dynamics. However, there are applications [4–6], where the distributed average tracking problem is relevant for designing distributed control laws for physical agents. In these applications, the dynamics of the physical agents must be taken into account in the control law design and the dynamics themselves introduce further challenges to the distributed average tracking problem. Distributed average tracking for physical agents with double-integrator dynamics and general linear dynamics are studied in, respectively, [14] and [15, 16]. In [17] a proportional-integral control scheme is extended to achieve distributed average tracking for physical Euler-Lagrange systems for two different kinds of input signals with steady states and with bounded derivatives.

It is noted that in the existing distributed average tracking algorithms employed for applications with physical double-integrator agents, both relative position and relative velocity measurements are required in the control laws [6, 14–17]. However, in practice, velocity measurements are usually less accurate and more expensive than position measurements. In addition, relative velocity measurements are often more challenging and expensive than absolute velocity measurements. We are hence motivated to solve the distributed average tracking problem for physical double-integrator agents with reduced requirement on velocity measurements, expanding on our preliminary work reported in [18]. In the context of distributed average tracking, reducing velocity measurements poses significant theoretical challenges. The reason is that unlike the consensus problem or the single-leader coordinated tracking problem where the leader has access to the tracking objective, there are significant additional inherent challenges in distributed average tracking as none of the agents has access to the tracking objective.

In this paper, two distributed algorithms (controller design combined with filter design) are introduced to achieve distributed average tracking with reduced requirement on velocity measurements and in the absence of correct position and velocity initialization. Each algorithm has its own relative benefits and is feasible for different application scenarios. In the first algorithm design, there is no need for either

absolute or relative velocity measurements. Each agent's algorithm employs its local relative positions with respect to neighbors, its own and neighbors' filter outputs accessed through communication and its own input signal, input velocity and input acceleration. The algorithm allows the agents to track the average of a wide class of time-varying input signals with bounded deviations among the input signals, among the input velocities, and among the input accelerations. Using this algorithm, distributed average tracking can be achieved in the absence of velocity measurements and correct initialization. In the second algorithm design, there is still no requirement for correct initialization and relative velocity measurements. Furthermore, inter-agent communication is not necessary and the algorithm can be implemented using only local sensing, which is desirable for certain applications (e.g., deep-space spacecraft formation flying), where communication might not be desirable or available. Each agent's algorithm only employs its local relative positions with respect to neighbors, its own velocity, input signal, input velocity and input acceleration. Distributed average tracking can be achieved provided that the input signals and their velocities and accelerations are all bounded.

Notations: Throughout the paper, $\mathbb{R}$ denotes the set of all real numbers and $\mathbb{R}^+$ the set of all positive real numbers. Let $\mathbf{1}_n$ and $\mathbf{0}_n$ denote the $n \times 1$ column vector of all ones and all zeros respectively. Let $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote, respectively, the maximum and minimum eigenvalues of a square real matrix with real eigenvalues. We use $\otimes$ to denote the Kronecker product, and $\text{sgn}(\cdot)$ to denote the signum function defined componentwise. For a vector function $x(t) : \mathbb{R} \mapsto \mathbb{R}^m$, define $\|x(t)\|_p$ as the p -norm, $x(t) \in \mathbb{L}_2$ if $\int_0^\infty x(\tau)^T x(\tau) d\tau < \infty$ and $x(t) \in \mathbb{L}_\infty$ if for each element of $x(t)$, $\sup_{t \geq 0} |x_i(t)| < \infty$, $i = 1, \dots, m$.

2 Problem Statement

Here, we consider n physical agents described by double-integrator dynamics

$$\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t), \quad i = 1, \dots, n, \quad (1)$$

where $x_i(t) \in \mathbb{R}^p$ and $v_i(t) \in \mathbb{R}^p$ are, respectively, i th agent's position and velocity, and $u_i(t)$ is its control input.

An *undirected* graph $G \triangleq (V, E)$ is used to characterize the interaction topology among the agents, where $V \triangleq \{1, \dots, n\}$ is the node set and $E \subseteq V \times V$ is the edge set. An edge $(j, i) \in E$ means that node i can obtain information from node j and vice versa. Self edges (i, i) are not considered here. N_i is the set of agents that are neighbors of agent i . Let m denote the number of edges in E , where the edges (j, i) and (i, j) are counted only once. The set of neighbors of node i is denoted as N_i . The *adjacency matrix* $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ of the graph G is defined such that the edge weight $a_{ij} = 1$ if $(j, i) \in E$ and $a_{ij} = 0$ otherwise. For an undirected graph, $a_{ij} = a_{ji}$. The *Laplacian*

matrix $L = [l_{ij}] \in \mathbb{R}^{n \times n}$ associated with $\mathbf{A}$ is defined as $l_{ii} = \sum_{j \neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$. For an undirected graph, L is symmetric positive semi-definite. By arbitrarily assigning an orientation for the edges in G , let $D \triangleq [d_{ij}] \in \mathbb{R}^{n \times m}$ be the *incidence matrix* associated with G , where $d_{ij} = -1$ if the edge e_j leaves node i , $d_{ij} = 1$ if it enters node i , and $d_{ij} = 0$ otherwise. The Laplacian matrix L is then given by $L = DD^T$ [19].

Assumption 2.1 *The undirected graph G is connected.*

Lemma 2.1 [19] *Under Assumption 2.1, the Laplacian matrix L has a simple zero eigenvalue such that $0 = \lambda_1(L) < \lambda_2(L) \leq \dots \leq \lambda_n(L)$, where $\lambda_i(\cdot)$ denotes the i th eigenvalue. Furthermore, for any vector $y \in \mathbb{R}^n$ satisfying $\mathbf{1}_n^T y = 0$, $\lambda_2(L)y^T y \leq y^T L y \leq \lambda_n(L)y^T y$.*

Suppose that each agent has a time-varying input signal $x_i^r(t) \in \mathbb{R}^p$, $i = 1, \dots, n$, satisfying

$$\dot{x}_i^r(t) = v_i^r(t), \quad \dot{v}_i^r(t) = a_i^r(t), \quad (2)$$

where $v_i^r(t) \in \mathbb{R}^p$ and $a_i^r(t) \in \mathbb{R}^p$ are, respectively, i th agent's input velocity and input acceleration. Here, we assume that the input signals are generated internally by the agents and each agent has access to its own input signal, input velocity, and input acceleration. Define $x(t) = [x_1^T, \dots, x_n^T]^T$, $v(t) = [v_1^T, \dots, v_n^T]^T$, $r(t) = [r_1^T, \dots, r_n^T]^T$, $v^r(t) = [v_1^{rT}, \dots, v_n^{rT}]^T$, and $a^r(t) = [a_1^{rT}, \dots, a_n^{rT}]^T$.

We study the distributed average tracking problem for the double-integrator agents. The goal is to design $u_i(t)$ for each agent to track the average of the input signals and input velocities, i.e., $\lim_{t \rightarrow \infty} \|x_i(t) - \frac{1}{n} \sum_{j=1}^n x_j^r(t)\|_2 = 0$, $\lim_{t \rightarrow \infty} \|v_i(t) - \frac{1}{n} \sum_{j=1}^n v_j^r(t)\|_2 = 0$, $i = 1, \dots, n$, where each agent has access to its own input information and has only local interaction with its neighbors. We are interested in controller design with reduced requirement on velocity measurements and in the absence of correct position and velocity initialization. First, we consider the scenario that each agent has communication capabilities but without the need for either absolute or relative velocity measurements. The agents achieve distributed average tracking for a wide class of input signals with bounded deviations among the input signals, among the input velocities, and among the input accelerations in the absence of velocity measurements and correct position and velocity initialization. Second, we consider the scenario that each agent has sensing but not necessarily communication capabilities without the need for relative velocity measurements. The agents achieve distributed average tracking in the absence of relative velocity measurements and correct position and velocity initialization.

3 Velocity free distributed average tracking in the absence of correct initialization

In this section, we consider the case where the agents can communicate with each other. To remove the requirement on velocity measurements, we introduce the following filter for agent i

$$\begin{aligned} \dot{\psi}_i(t) &= \kappa(x_i(t) - x_i^r(t)) - 2\kappa(\omega_i(t) + v_i^r(t)) \\ &\quad + \alpha \sum_{j \in N_i} [x_i(t) - x_j(t)] - \alpha^2 \sum_{j \in N_i} [\omega_i(t) - \omega_j(t)] \\ &\quad - \gamma \sum_{j \in N_i} \text{sgn}[\omega_i(t) - \omega_j(t)] - a_i^r(t), \end{aligned} \quad (3)$$

$$\omega_i(t) = \psi_i(t) - \kappa(x_i(t) - x_i^r(t)) - \alpha \sum_{j \in N_i} [x_i(t) - x_j(t)], \quad (4)$$

where $\psi_i \in \mathbb{R}^p$ is an auxiliary filter variable, $\omega_i \in \mathbb{R}^p$ is the filter output, and κ , α , and $\gamma \in \mathbb{R}^+$ are control gains to be designed.

We propose the following distributed control law for agent i

$$\begin{aligned} u_i(t) &= -\kappa(x_i(t) - x_i^r(t)) + \kappa(\omega_i(t) + v_i^r(t)) \\ &\quad - \alpha\beta \sum_{j \in N_i} [x_i(t) - x_j(t)] + \alpha^2 \sum_{j \in N_i} [\omega_i(t) - \omega_j(t)] \\ &\quad + \gamma \sum_{j \in N_i} \text{sgn}[\omega_i(t) - \omega_j(t)] + a_i^r(t), \end{aligned} \quad (5)$$

where $\beta \in \mathbb{R}^+$ is another control gain to be designed. For notational simplicity, we will remove the index t from variables in the remainder of the paper. Here, $x_i - x_i^r$ and $\sum_{j \in N_i} (x_i - x_j)$ are filtered through the first-order filter (4). The relative filter output $\omega_i - \omega_j$ with respect to neighbors is employed to substitute the absolute and relative velocity measurements, v_i and $\sum_{j \in N_i} (v_i - v_j)$. The term $\sum_{j \in N_i} (x_i - x_j)$ is employed in (5) to drive the agents' positions to consensus. The terms $x_i - x_i^r$, $\omega_i + v_i^r$, and a_i^r in (5) are used to guarantee that $\sum_{j=1}^n x_j \rightarrow \sum_{j=1}^n x_j^r$ and $\sum_{j=1}^n v_j \rightarrow \sum_{j=1}^n v_j^r$ in the absence of correct position and velocity initialization. The term $\sum_{j \in N_i} \text{sgn}(\omega_i - \omega_j)$ is employed in (5) to determine how much effort and in which direction agent i should adopt to guarantee consensus in the presence of x_i^r , v_i^r , and a_i^r . The terms in (3) are designed accordingly to carry out the proof as shown later. To implement (3)-(5), there is no need for either absolute or relative velocity measurements but the communication is necessary between neighbors. Since each agent needs its neighbors' filter outputs, which cannot be measured by the agent itself.

Assumption 3.1 The deviations among input signals, among input velocities, and among input accelerations are bounded, i.e., $\sup_{t \in [0, \infty)} \|x_i^r(t) - x_j^r(t)\|_2 \leq \bar{r}_d$, $\sup_{t \in [0, \infty)} \|v_i^r(t) - v_j^r(t)\|_2 \leq \bar{v}_d^r$, $\sup_{t \in [0, \infty)} \|a_i^r(t) - a_j^r(t)\|_2 \leq \bar{a}_d^r$, $i, j = 1, \dots, n$, where $\bar{r}_d, \bar{v}_d^r, \bar{a}_d^r \in \mathbb{R}^+$.

Theorem 3.2 Using the control law given by (3)-(5) for system (1), distributed average tracking is achieved asymptotically, provided that Assumptions 2.1 and 3.1 hold and the gains κ, β, α , and γ are chosen such that $\kappa > 0$, $\beta > 1$, $\alpha > \frac{1}{\lambda_2(L)} + \max\{\frac{\lambda_{\max}^2(L)}{\lambda_2^2(L)}, \frac{3\lambda_{\max}(L)-1}{\lambda_2(L)}, \frac{1}{12\kappa\lambda_2(L)}\}$, and $\gamma > n(n-1)(\kappa\bar{r}_d + \kappa\bar{v}_d^r + \bar{a}_d^r)$, where $\lambda_2(L)$ and $\lambda_n(L)$ are defined in Lemma 2.1.

Proof: The proof contains two steps. First, we prove for each agent $x_i \rightarrow \frac{1}{n} \sum_{j=1}^n x_j$ and $v_i \rightarrow \frac{1}{n} \sum_{j=1}^n v_j$. Then by proving that $\sum_{j=1}^n x_j \rightarrow \sum_{j=1}^n x_j^r$ and $\sum_{j=1}^n v_j \rightarrow \sum_{j=1}^n v_j^r$, it can be concluded that $x_i \rightarrow \frac{1}{n} \sum_{j=1}^n x_j^r$ and $v_i \rightarrow \frac{1}{n} \sum_{j=1}^n v_j^r$, $i = 1, \dots, n$, and hence distributed average tracking is achieved.

First, by using the control law (5) for (1), we can get

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= -\kappa(x_i - x_i^r) + \kappa(\omega_i + v_i^r) \\ &\quad - \alpha\beta \sum_{j \in N_i} [x_i - x_j] + \alpha^2 \sum_{j \in N_i} [\omega_i - \omega_j] \\ &\quad + \gamma \sum_{j \in N_i} \text{sgn}[\omega_i - \omega_j] + a_i^r. \end{aligned} \quad (6)$$

Due to the existence of the signum function in the algorithm, the closed-loop dynamics is discontinuous. Thus, the solutions should be investigated in terms of differential inclusions by using nonsmooth analysis [20,21]. Since the signum function is measurable and locally essentially bounded, it is concluded that the Filippov solutions for the closed-loop dynamics (6) always exist and are absolutely continuous [22].

Define $\xi_x = [M \otimes I_p]x$, $\xi_v = [M \otimes I_p]v$, $\xi_\omega = [M \otimes I_p]\omega$ and $\xi = [\xi_x^T \ \xi_v^T \ \xi_\omega^T]^T$, where $M = I_n - \frac{1}{n} \mathbf{1}_n \mathbf{1}_n^T$ and $\omega = [\omega_1^T, \dots, \omega_n^T]^T$. It follows from (3), (4), and (6) that

$$\dot{\xi} \in^{a.e.} \mathcal{K}[f](\xi) \quad (7)$$

where $\mathcal{K}$ denotes the Filippov set-valued map, *a.e.* stands for “almost everywhere”, $f = [f_x^T \ f_v^T \ f_\omega^T]^T$ and

$$\begin{aligned} f_x(\xi) &= \xi_v, \\ f_v(\xi) &= -\kappa\xi_x + \kappa(M \otimes I_p)r + \kappa\xi_\omega + \kappa(M \otimes I_p)v^r \\ &\quad - \alpha\beta(L \otimes I_p)\xi_x + \alpha^2(L \otimes I_p)\xi_\omega \\ &\quad + \gamma(D \otimes I_p)\text{sgn}[(D^T \otimes I_p)\xi_\omega] + (M \otimes I_p)a^r, \end{aligned}$$

$$\begin{aligned} f_\omega(\xi) &= \kappa\xi_x - \kappa(M \otimes I_p)r - 2\kappa\xi_\omega - \kappa(M \otimes I_p)v^r \\ &\quad + \alpha(L \otimes I_p)\xi_x - \alpha^2(L \otimes I_p)\xi_\omega \\ &\quad - \gamma(D \otimes I_p)\text{sgn}[(D^T \otimes I_p)\xi_\omega] - (M \otimes I_p)a^r - \kappa\xi_v \\ &\quad - \alpha(L \otimes I_p)\xi_v, \end{aligned}$$

where we have used $M \times M = M$, $L \times M = L$ and $D \times M = D$ to obtain $f_x(\cdot)$, $f_v(\cdot)$ and $f_\omega(\cdot)$.

$$\text{Consider the function } V = \frac{1}{2}\xi^T \begin{bmatrix} \mu[L \otimes I_p] + \kappa I_{np} & I_{np} & I_{np} \\ I_{np} & I_{np} & I_{np} \\ I_{np} & I_{np} & \beta I_{np} \end{bmatrix} \xi,$$

where $\mu \in \mathbb{R}^+$. Since $\xi_x^T \mathbf{1}_{np} = 0$, using Lemma 2.1,

$$\text{we have } V \geq \frac{1}{2}\xi^T \begin{bmatrix} \mu\lambda_2(L) + \kappa & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & \beta \end{bmatrix} \otimes I_{np} \xi. \text{ It can}$$

be proved that if $\mu > \frac{1}{\lambda_2(L)}$, $\kappa > 0$, and $\beta > 1$, then

$$\begin{bmatrix} \mu\lambda_2(L) + \kappa & 1 \\ 1 & 1 \end{bmatrix} - \frac{1}{\beta} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} > 0. \text{ Thus, using Shur}$$

complement, it is concluded that V is positive definite. Since V is a continuous function, its set-valued Lie derivative, $\dot{V}$, along (7) is given as

$$\begin{aligned} \dot{V} &= \mathcal{K} \left[\mu\xi_x^T(L \otimes I_p)\xi_v^T - \kappa\xi_x^T\xi_\omega - \alpha\beta\xi_x^T(L \otimes I_p)\xi_x \right. \\ &\quad + \alpha\xi_x^T(L \otimes I_p)\xi_x + \xi_v^T\xi_v - \kappa\xi_v^T\xi_\omega - \alpha\beta\xi_v^T(L \otimes I_p)\xi_x \\ &\quad - \kappa\xi_v^T\xi_v - \alpha\xi_v^T(L \otimes I_p)\xi_v + \xi_\omega^T\xi_\omega - \kappa\xi_\omega^T\xi_x \\ &\quad + \kappa\xi_\omega^T(M \otimes I_p)r + \kappa\xi_\omega^T\xi_\omega + \kappa\xi_\omega^T(M \otimes I_p)v^r \\ &\quad + \alpha^2\xi_\omega^T(L \otimes I_p)\xi_\omega + \gamma\xi_\omega^T(D \otimes I_p)\text{sgn}[(D^T \otimes I_p)\xi_\omega] \\ &\quad + \xi_\omega^T(M \otimes I_p)a^r + \beta\kappa\xi_\omega^T\xi_x - \beta\kappa\xi_\omega^T(M \otimes I_p)r - 2\beta\kappa\xi_\omega^T\xi_\omega \\ &\quad - \beta\kappa\xi_\omega^T(M \otimes I_p)v^r - \alpha^2\beta\xi_\omega^T(L \otimes I_p)\xi_\omega \\ &\quad - \beta\gamma\xi_\omega^T(D \otimes I_p)\text{sgn}[(D^T \otimes I_p)\xi_\omega] - \beta\xi_\omega^T(M \otimes I_p)a^r \\ &\quad \left. - \beta\kappa\xi_\omega^T\xi_v - \alpha\beta\xi_\omega^T(L \otimes I_p)\xi_v \right] \\ &= \left\{ \mu\xi_x^T(L \otimes I_p)\xi_v^T - \kappa\xi_x^T\xi_\omega - \alpha\beta\xi_x^T(L \otimes I_p)\xi_x \right. \\ &\quad + \alpha\xi_x^T(L \otimes I_p)\xi_x + \xi_v^T\xi_v - \kappa\xi_v^T\xi_\omega - \alpha\beta\xi_v^T(L \otimes I_p)\xi_x \\ &\quad - \kappa\xi_v^T\xi_v - \alpha\xi_v^T(L \otimes I_p)\xi_v + \xi_\omega^T\xi_\omega - \kappa\xi_\omega^T\xi_x + \kappa\xi_\omega^T(M \otimes I_p)r \\ &\quad + \kappa\xi_\omega^T\xi_\omega + \kappa\xi_\omega^T(M \otimes I_p)v^r + \alpha^2\xi_\omega^T(L \otimes I_p)\xi_\omega \\ &\quad + \gamma\xi_\omega^T(D \otimes I_p)\text{sgn}[(D^T \otimes I_p)\xi_\omega] + \xi_\omega^T(M \otimes I_p)a^r \\ &\quad + \beta\kappa\xi_\omega^T\xi_x - \beta\kappa\xi_\omega^T(M \otimes I_p)r - 2\beta\kappa\xi_\omega^T\xi_\omega - \beta\kappa\xi_\omega^T(M \otimes I_p)v^r \\ &\quad - \alpha^2\beta\xi_\omega^T(L \otimes I_p)\xi_\omega - \beta\gamma\xi_\omega^T(D \otimes I_p)\text{sgn}[(D^T \otimes I_p)\xi_\omega] \\ &\quad \left. - \beta\xi_\omega^T(M \otimes I_p)a^r - \beta\kappa\xi_\omega^T\xi_v - \alpha\beta\xi_\omega^T(L \otimes I_p)\xi_v \right\}, \end{aligned} \quad (8)$$

where $(M \otimes I_p)\xi_x = (M^2 \otimes I_p)x = (M \otimes I_p)x = \xi_x$ and using the same analysis $(M \otimes I_p)\xi_v = \xi_v$ and $(M \otimes I_p)\xi_\omega = \xi_\omega$. Also we have used the fact that $\mathcal{K}[\xi_\omega^T(D \otimes I_p)\text{sgn}[(D^T \otimes$

$I_p)\xi_\omega\big] = \mathcal{K}\big[\|(D^T \otimes I_p)\xi_\omega\|_1\big] = \big\{\|(D^T \otimes I_p)\xi_\omega\|_1\big\}$, since $\|(D^T \otimes I_p)\xi_\omega\|_1$ is continuous during the whole time. We can analyze the term $\xi_\omega^T(M \otimes I_p)(\kappa r + \kappa v^r + a^r)$ as

$$\begin{aligned} \xi_\omega^T(M \otimes I_p)(\kappa r + \kappa v^r + a^r) &= \xi_\omega^T(M^2 \otimes I_p)(\kappa r + \kappa v^r + a^r) \\ &\leq \|(M \otimes I_p)\xi_\omega\|_2 \|(M \otimes I_p)(\kappa r + \kappa v^r + a^r)\|_2 \\ &\leq (\kappa \bar{r}_d + \kappa \bar{v}_d^r + \bar{a}_d^r) \sum_{i=1}^n \sum_{j=1, j \neq i}^n \|\xi_{\omega i} - \xi_{\omega j}\|_2 \\ &\leq (\kappa \bar{r}_d + \kappa \bar{v}_d^r + \bar{a}_d^r) \sum_{i=1}^n \max_i \left\{ \sum_{j=1, j \neq i}^n \|\xi_{\omega i} - \xi_{\omega j}\|_2 \right\} \\ &\leq n(\kappa \bar{r}_d + \kappa \bar{v}_d^r + \bar{a}_d^r) \max_i \left\{ \sum_{j=1, j \neq i}^n \|\xi_{\omega i} - \xi_{\omega j}\|_2 \right\} \\ &\leq \frac{n(n-1)}{2} (\kappa \bar{r}_d + \kappa \bar{v}_d^r + \bar{a}_d^r) \sum_{i=1}^n \sum_{j \in N_i} \|\xi_{\omega i} - \xi_{\omega j}\|_2 \\ &\leq \frac{n(n-1)}{2} (\kappa \bar{r}_d + \kappa \bar{v}_d^r + \bar{a}_d^r) \sum_{i=1}^n \sum_{j \in N_i} \|\xi_{\omega i} - \xi_{\omega j}\|_1, \end{aligned} \quad (9)$$

where we have used Assumption 3.1 and $\|\cdot\|_2 \leq \|\cdot\|_1$ to obtain the second and the last inequalities, respectively. Let $\mu = \alpha\beta$. Therefore, $\alpha > \frac{1}{\lambda_2(L)}$ and $\beta > 1$ ensures that $\mu > \frac{1}{\lambda_2(L)}$. Note that $\dot{V}$ is a singleton and the function V is continuously differentiable.

Combining (8) and (9), it follows from $\dot{V} \in \dot{\hat{V}}$, where $\dot{V}$ denotes the derivative of V , that

$$\begin{aligned} \dot{V} &\leq \kappa(\beta - 2)\xi_x^T \xi_\omega - \alpha(\beta - 1)\lambda_2(L)\xi_x^T \xi_x \\ &\quad + \left(1 - \kappa - \alpha\lambda_2(L)\right)\xi_v^T \xi_v^T \\ &\quad + \xi_v^T \left((1 - \kappa - \kappa\beta)I_{np} - \alpha\beta(L \otimes I_p)\right)\xi_\omega \\ &\quad + \frac{n(n-1)(\beta-1)(\kappa \bar{r}_d + \kappa \bar{v}_d^r + \bar{a}_d^r)}{2} \times \\ &\quad \sum_{i=1}^n \sum_{j \in N_i} \|\xi_{\omega i} - \xi_{\omega j}\|_1 \\ &\quad + \left(\kappa - 2\kappa\beta + \alpha^2(1 - \beta)\lambda_2(L)\right)\xi_\omega^T \xi_\omega \\ &\quad - \gamma(\beta - 1) \sum_{i=1}^n \sum_{j \in N_i} \xi_{\omega i}^T \text{sgn}(\xi_{\omega i} - \xi_{\omega j}) \\ &= \kappa(\beta - 2)\xi_x^T \xi_\omega - \alpha(\beta - 1)\lambda_2(L)\xi_x^T \xi_x \\ &\quad + \left(1 - \kappa - \alpha\lambda_2(L)\right)\xi_v^T \xi_v^T \\ &\quad + \xi_v^T \left((1 - \kappa - \kappa\beta)I_{np} - \alpha\beta(L \otimes I_p)\right)\xi_\omega \\ &\quad + \frac{n(n-1)(\beta-1)(\kappa \bar{r}_d + \kappa \bar{v}_d^r + \bar{a}_d^r)}{2} \times \end{aligned}$$

$$\begin{aligned} &\sum_{i=1}^n \sum_{j \in N_i} \|\xi_{\omega i} - \xi_{\omega j}\|_1 \\ &\quad + \left(\kappa - 2\kappa\beta + \alpha^2(1 - \beta)\lambda_2(L)\right)\xi_\omega^T \xi_\omega \\ &\quad - \frac{\gamma(\beta - 1)}{2} \sum_{i=1}^n \sum_{j \in N_i} (\xi_{\omega i} - \xi_{\omega j})^T \text{sgn}(\xi_{\omega i} - \xi_{\omega j}), \end{aligned}$$

where we have used $\xi_\omega^T(D \otimes I_p)\text{sgn}[(D^T \otimes I_p)\xi_\omega] = \sum_{i=1}^n \sum_{j \in N_i} \xi_{\omega i}^T \text{sgn}(\xi_{\omega i} - \xi_{\omega j})$ and Lemma 2.1 to obtain the first inequality. The gain β should satisfy the inequality $\beta > 1$; however, to make the proof easier, here we choose $\beta = 2$ without loss of generality. Thus, if $\gamma > n(n-1)(\kappa \bar{r}_d + \kappa \bar{v}_d^r + \bar{a}_d^r)$, we will have $\dot{V} \leq \xi^T Q \xi$, where $Q \triangleq [Q_{ij}]$, $i, j = 1, \dots, 3$, $Q_{ij} \in \mathbb{R}^{np \times np}$, $Q_{11} = -\alpha\lambda_2(L)I_{np}$, $Q_{22} = [1 - \kappa - \alpha\lambda_2(L)]I_{np}$, $Q_{23} = Q_{32} = \frac{1-3\kappa}{2}I_{np} - \alpha[L \otimes I_p]$, $Q_{33} = -[3\kappa + \alpha^2\lambda_2(L)]I_{np}$ and $Q_{12} = Q_{13} = Q_{21} = Q_{31} = \mathbf{0}_{np}$. Therefore, if α satisfies the condition $\alpha > \frac{1}{\lambda_2(L)} + \max\left\{\frac{\lambda_{\max}^2(L)}{\lambda_2^2(L)}, \frac{3\lambda_{\max}(L)-1}{\lambda_2(L)}, \frac{1}{12\kappa\lambda_2(L)}\right\}$, it can be seen that $\dot{V}$ is negative definite. Using Theorem 3.1 in [23], it is concluded that $\xi = \mathbf{0}_{3np}$ is globally asymptotically stable, which means $x_i \rightarrow \frac{1}{n} \sum_{j=1}^n x_j$, $v_i \rightarrow \frac{1}{n} \sum_{j=1}^n v_j$, and $\omega_i \rightarrow \frac{1}{n} \sum_{j=1}^n \omega_j$, $i = 1, \dots, n$.

Second, define the variables $S_x = \sum_{j=1}^n (x_j - x_j^r)$,

$S_v = \sum_{j=1}^n (v_j - v_j^r)$, and $S_\omega = \sum_{j=1}^n (\omega_j + v_j^r)$. Then, we can get

$$\text{from (3)-(4) and (6) that } \dot{S} = \begin{pmatrix} 0 & 1 & 0 \\ -\kappa & 0 & \kappa \\ \kappa & -\kappa & -2\kappa \end{pmatrix} \otimes I_p S =$$

$(A \otimes I_p)S$, where $S = [S_x^T, S_v^T, S_\omega^T]^T$. If $\kappa > 0$, the matrix A is Hurwitz. Therefore, $\lim_{t \rightarrow \infty} S = \mathbf{0}_p$, which means $\lim_{t \rightarrow \infty} \sum_{j=1}^n x_j = \sum_{j=1}^n x_j^r$ and $\lim_{t \rightarrow \infty} \sum_{j=1}^n v_j = \sum_{j=1}^n v_j^r$.

Combining the two parts shows that $x_i \rightarrow \frac{1}{n} \sum_{j=1}^n x_j^r$ and $v_i \rightarrow \frac{1}{n} \sum_{j=1}^n v_j^r$ asymptotically. Therefore, distributed average tracking is achieved asymptotically. ■

Remark 3.3 Compared with [14], using the algorithm defined by (3)-(5), the requirement for both absolute and relative velocity measurements is removed, which could lower the cost of the system and save space by completely removing the velocity sensors. In addition, the algorithm in [14] relies on correct initialization. While, the introduced algorithm (3)-(5) is robust to initialization errors.

Remark 3.4 Here, the discontinuous signum function, which switches instantaneously at 0, is used to compensate for input signals, input velocities and input accelerations for each agent so that the agents can reach consensus. However, by employing the signum function in the control

law, there might be chattering in each agent's states in practice. To reduce the effects of chattering caused by the discontinuous control actions, many analytical design methods have been proposed. The boundary layer approach is one of the most cited approaches among the existing methods [24, 25], which approximates the switching function by a continuous (or smooth) function such as $\frac{z}{\|z\|_2 + \epsilon e^{-\phi t}}$, $\text{sat}(\frac{z}{\epsilon})$ or $\tanh(\frac{z}{\epsilon})$, where $z \in \mathbb{R}^n$, ϵ and $\phi \in \mathbb{R}^+$. In our simulation, we have used the function $h(z) = \frac{z}{\|z\|_2 + \epsilon e^{-\phi t}}$ as a smooth approximation of the signum function. The size of the boundary layer, $\epsilon e^{-\phi t}$ is time-varying and as $t \rightarrow \infty$ the continuous function $h(z)$ approaches the signum function. It can be proved that by replacing the signum function with the continuous function $h(\cdot)$ the distributed average tracking can still be achieved with zero error. Due to space limitation, we omit the details.

4 Distributed average tracking in the absence of relative velocity information and communication

In the previous section, the proposed algorithm solves the distributed average tracking problem in the presence of communication without velocity measurements. In this section, we deal with the distributed average tracking problem without communication in the absence of the relative velocity information and correct initialization constraint.

Assumption 4.1 Each input signal, its velocity and acceleration are bounded, i.e., $\sup_{t \in [0, \infty)} \|x_i^r(t)\|_2 \leq \bar{r}$, $\sup_{t \in [0, \infty)} \|v_i^r(t)\|_2 \leq \bar{v}^r$ and $\sup_{t \in [0, \infty)} \|a_i^r(t)\|_2 \leq \bar{a}^r$, $i = 1, \dots, n$, where $\bar{r}$, $\bar{v}^r$, $\bar{a}^r \in \mathbb{R}^+$.

We introduce the following filter for agent i

$$\dot{\psi}_i = \alpha \sum_{j \in N_i} (x_i - x_j) - \alpha^2 \omega_i - \kappa x_i^r - \kappa v_i^r - a_i^r - \gamma \text{sgn}(\omega_i), \quad (10)$$

$$\omega_i = \psi_i - \alpha \sum_{j \in N_i} (x_i - x_j), \quad (11)$$

where $\psi_i \in \mathbb{R}^p$ is an auxiliary filter variable, $\omega_i \in \mathbb{R}^p$ is the filter output, and κ , α , $\gamma \in \mathbb{R}^+$ are control gains.

We propose the following control input for agent i

$$u_i = -\kappa(x_i - x_i^r) - \kappa(v_i - v_i^r) - \alpha\beta \sum_{j \in N_i} (x_i - x_j) + \alpha^2 \omega_i + \gamma \text{sgn}(\omega_i) + a_i^r, \quad (12)$$

where $\beta \in \mathbb{R}^+$ is another control gain to be designed. In (12), the terms $\kappa(x_i - x_i^r)$, $\kappa(v_i - v_i^r)$ and a_i^r are used to drive the two errors $\sum_{j=1}^n (x_j - x_j^r)$ and $\sum_{j=1}^n (v_j - v_j^r)$ to zero because the correct initialization condition does not hold. The term $\text{sgn}(\omega_i)$ is employed to guarantee consensus in the presence

of x_i^r , v_i^r and a_i^r in each agent's closed-loop dynamics. To implement (10)-(12), there is no need for relative velocity measurements between neighbors and the relative positions are the only information related to neighbors, which can be obtained by using only local sensing. Therefore, communication between neighbors is not necessary. The following theorem presents sufficient conditions to solve distributed average tracking without neighbors' velocity and communication.

Theorem 4.2 Distributed average tracking is achieved for system (1) using (10)-(12) asymptotically, under Assumptions 2.1 and 4.1 and provided that the gains κ , β , α , and γ are chosen such that $\kappa > 0$, $\beta > \sqrt{n}$, $\gamma > \frac{1+\beta}{\beta-\sqrt{n}}(\kappa\bar{r} + \kappa\bar{v}^r + \bar{a}^r)$, $\alpha > \frac{1}{\lambda_2(L)} + \max\{\frac{\beta^2 \lambda_{\max}^2(L)}{4(\beta-1)\lambda_2(L)}, \frac{\beta \lambda_{\max}(L)(\kappa-1)}{2\kappa}, \frac{2\kappa^2}{\lambda_2(L)}\}$, where $\lambda_2(L)$ and $\lambda_n(L)$ are defined in Lemma 2.1.

Proof: Similar to Section 3, the proof contains two steps. In the first step, it is proved that $x_i \rightarrow \frac{1}{n} \sum_{j=1}^n x_j$, $v_i \rightarrow \frac{1}{n} \sum_{j=1}^n v_j$, and $\omega_i \rightarrow 0$ asymptotically. Again due to the existence of the signum function in the algorithm, the closed-loop dynamics is discontinuous. Thus, the solutions should be investigated in terms of differential inclusions by using nonsmooth analysis. Define ξ_x , ξ_v , M , and ω as in the proof of Theorem 3.2. Let $\xi' = [\xi_x^T \ \xi_v^T \ \omega^T]^T$ and $f' = [f_x^T \ f_v^T \ f_\omega^T]^T$. By rewriting the dynamics (1) using (12) in vector form, it follows from (11) that

$$\dot{\xi}' \in \mathcal{K}[f'](\xi') \quad (13)$$

where

$$\begin{aligned} f_x' &= \xi_v, \\ f_v' &= -\kappa \xi_x + \kappa(M \otimes I_p)r - \kappa \xi_v + \kappa(M \otimes I_p)v^r \\ &\quad - \alpha\beta[L \otimes I_p]\xi_x + \alpha^2(M \otimes I_p)\omega + \gamma(M \otimes I_p)\text{sgn}(\omega) \\ &\quad + (M \otimes I_p)a^r, \\ f_\omega' &= \alpha[L \otimes I_p]\xi_x - \alpha^2\omega - \kappa r - \kappa v^r - a^r - \gamma \text{sgn}(\omega) \\ &\quad - \alpha[L \otimes I_p]\xi_v. \end{aligned}$$

$$\text{Consider the function } V = \frac{1}{2} \xi'^T \begin{bmatrix} \mu[L \otimes I_p] + 2\kappa I_{np} & I_{np} & I_{np} \\ I_{np} & I_{np} & I_{np} \\ I_{np} & I_{np} & \beta I_{np} \end{bmatrix} \xi',$$

where $\mu \in \mathbb{R}^+$. Using the same analysis as in Section 3, if $\mu > \frac{1}{\lambda_2(L)}$, $\kappa > 0$, and $\beta > 1$, the function V is positive definite. Since V is a continuous function, its set-valued Lie derivative along (13) is given as

$$\begin{aligned} \dot{V} &= \mathcal{K} \left[\mu \xi_x^T [L \otimes I_p] \xi_v - \kappa \xi_x^T \xi_x - \alpha \beta \xi_x^T [L \otimes I_p] \xi_x \right. \\ &\quad + \alpha \xi_x^T [L \otimes I_p] \xi_x + \xi_v^T \xi_v - \kappa \xi_v^T \xi_v - \alpha \beta \xi_v^T [L \otimes I_p] \xi_x \\ &\quad \left. - \alpha \xi_v^T [L \otimes I_p] \xi_v + \omega^T \xi_v - \kappa \omega^T \xi_x + \kappa \omega^T (M \otimes I_p) r \right] \end{aligned}$$

$$\begin{aligned}
& -\kappa\omega^T\xi_v + \kappa\omega^T(M \otimes I_p)v^r + \alpha^2\omega^T(M \otimes I_p)\omega \\
& + \gamma\omega^T(M \otimes I_p)\text{sgn}(\omega) + \omega^T(M \otimes I_p)a^r \\
& - \alpha^2\beta\omega^T\omega - \beta\kappa\omega^Tr - \beta\kappa\omega^Tv^r - \beta\omega^Ta^r \\
& - \beta\gamma\omega^T\text{sgn}(\omega) - \alpha\beta\omega^T[L \otimes I_p]\xi_v \Big] \\
& = \left\{ \mu\xi_x^T[L \otimes I_p]\xi_v - \kappa\xi_x^T\xi_x - \alpha\beta\xi_x^T[L \otimes I_p]\xi_x \right. \\
& + \alpha\xi_x^T[L \otimes I_p]\xi_x + \xi_v^T\xi_v - \kappa\xi_v^T\xi_v - \alpha\beta\xi_v^T[L \otimes I_p]\xi_x \\
& - \alpha\xi_v^T[L \otimes I_p]\xi_v + \omega^T\xi_v - \kappa\omega^T\xi_x + \kappa\omega^T(M \otimes I_p)r \\
& - \kappa\omega^T\xi_v + \kappa\omega^T(M \otimes I_p)v^r + \alpha^2\omega^T(M \otimes I_p)\omega \\
& + \gamma\omega^T(M \otimes I_p)\text{sgn}(\omega) + \omega^T(M \otimes I_p)a^r \\
& - \alpha^2\beta\omega^T\omega - \beta\kappa\omega^Tr - \beta\kappa\omega^Tv^r - \beta\omega^Ta^r \\
& \left. - \beta\gamma\omega^T\text{sgn}(\omega) - \alpha\beta\omega^T[L \otimes I_p]\xi_v \right\}, \tag{14}
\end{aligned}$$

where, we used the fact that $\mathcal{K}[\omega^T\text{sgn}(\omega)] = \mathcal{K}[\|\omega\|_1] = \{\|\omega\|_1\}$. Here,

$$\begin{aligned}
& \kappa\omega^T(M \otimes I_p)r + \kappa\omega^T(M \otimes I_p)v^r + \omega^T(M \otimes I_p)a^r \\
& + \gamma\omega^T(M \otimes I_p)\text{sgn}(\omega) - \beta\kappa\omega^Tr - \beta\kappa\omega^Tv^r - \beta\omega^Ta^r \\
& - \beta\gamma\omega^T\text{sgn}(\omega) \\
& = \omega^T[(M - \beta I_n) \otimes I_p] \left[\kappa r + \kappa v^r + a^r + \gamma\text{sgn}(\omega) \right] \\
& \leq [(1 + \beta)(\kappa\bar{r} + \kappa\bar{v}^r + a^r) + \gamma\sqrt{n}] \cdot \|\omega\|_2 - \beta\gamma\|\omega\|_1 \\
& \leq [(1 + \beta)(\kappa\bar{r} + \kappa\bar{v}^r + a^r) + \gamma\sqrt{n}] \cdot \|\omega\|_1 - \beta\gamma\|\omega\|_1. \tag{15}
\end{aligned}$$

Combining (14) and (15), if $\beta > \sqrt{n}$, $\gamma > \frac{1+\beta}{\beta-\sqrt{n}}(\kappa\bar{r} + \kappa\bar{v}^r + a^r)$ and $\mu = \alpha\beta$, it follows from $\dot{V} \in \dot{V}$ that

$$\begin{aligned}
\dot{V} & \leq -\kappa\xi_x^T\xi_x - \alpha(\beta - 1)\lambda_2(L)\xi_x^T\xi_x + \xi_v^T\xi_v - \kappa\xi_v^T\xi_v \\
& - \alpha\lambda_2(L)\xi_v^T\xi_v + \omega^T\xi_v - \kappa\omega^T\xi_x - \kappa\omega^T\xi_v \\
& + \alpha^2\omega^T(M \otimes I_p)\omega - \alpha^2\beta\omega^T\omega - \alpha\beta\omega^T[L \otimes I_p]\xi_v \\
& = \xi'^T P \xi',
\end{aligned}$$

where $P \triangleq [P_{ij}]$, $i, j = 1, \dots, 3$, $P_{ij} \in \mathbb{R}^{np \times np}$, $P_{11} = -[\kappa + \alpha(\beta - 1)\lambda_2(L)]I_{np}$, $P_{22} = [1 - \kappa - \alpha\lambda_2(L)]I_{np}$, $P_{12} = P_{21} = \mathbf{0}_{np}$, $P_{13} = P_{31} = -\frac{1}{2}\kappa I_{np}$, $P_{23} = P_{32} = \frac{1}{2}[(1 - \kappa)I_{np} - \alpha\beta L]$, and $P_{33} = -\alpha^2(\beta - 1)I_{np}$ and we have used the fact that $M < I_n$ and Lemma 2.1 to obtain the inequality. If the control gains κ , α , and β satisfy the constraints mentioned in Theorem 4.2, the matrix P and thus $\dot{V}$ are negative definite. Similar to the proof of Theorem 3.2, by using Theorem 3.1 in [23], it is concluded that $\xi' = \mathbf{0}_{3np}$ is globally asymptotically stable, which implies that $x_i - \frac{1}{n}\sum_{j=1}^n x_j \rightarrow 0$, $v_i - \frac{1}{n}\sum_{j=1}^n v_j \rightarrow 0$ and $\omega_i \rightarrow 0$ as $t \rightarrow \infty$.

In the second step, we prove that $\sum_{j=1}^n x_j \rightarrow \sum_{j=1}^n x_j^r$ and $\sum_{j=1}^n v_j \rightarrow \sum_{j=1}^n v_j^r$ asymptotically. Define S_x , and S_v as

in the proof of Theorem 3.2. Then, we can get from (2) and (12) that

$$\begin{aligned}
\dot{S}_x & = S_v, \\
\dot{S}_v & = -\kappa S_x - \kappa S_v + \alpha^2 \sum_{i=1}^n \omega_i + \gamma \sum_{i=1}^n \text{sgn}(\omega_i). \tag{16}
\end{aligned}$$

We then use input-to-state stability to analyze the system (16) by treating the term $\alpha^2 \sum_{i=1}^n \omega_i + \gamma \sum_{i=1}^n \text{sgn}(\omega_i)$ as the input and S_x and S_v as the states. If $\kappa > 0$, the matrix $\begin{bmatrix} \mathbf{0}_p & I_p \\ -\kappa I_p & -\kappa I_p \end{bmatrix}$ is Hurwitz. The system (16) with zero input is exponentially stable and hence input-to-state stable. Since $\omega_i \rightarrow 0$ as $t \rightarrow \infty$ for each agent, it follows that $S_x \rightarrow 0$ and $S_v \rightarrow 0$, which implies that $\sum_{j=1}^n x_j \rightarrow \sum_{j=1}^n x_j^r$ and $\sum_{j=1}^n v_j \rightarrow \sum_{j=1}^n v_j^r$, respectively. Employing the result of first step, it is concluded that $x_i \rightarrow \frac{1}{n}\sum_{j=1}^n x_j^r$ and $v_i \rightarrow \frac{1}{n}\sum_{j=1}^n v_j^r$ and the distributed average tracking is achieved. ■

Remark 4.3 Both algorithms in Sections 3 and 4 have their unique features and advantages while with trade-offs. Depending on the application scenarios, one might be more appropriate than the other. The advantages of the algorithm (3)-(5) are that it can deal with more general input signals, and both absolute and relative velocity measurements are removed. However, the trade-off of the algorithm is that it necessitates communication between neighbors. Note that if the agents' positions and velocities satisfy an initialization condition, the algorithm can be easily modified to deal with a even larger group of input signals. It should be noted that, this assumption is not very restrictive for some applications. In fact, the setup of the distributed average tracking problem is that each agent knows its own input signal and input velocity accurately. Even for the simpler static distributed averaging problem (computing the average of constant values), each agent still needs to know its own quantity to be averaged accurately. Otherwise, it is hard to imagine how the problem can even be solved. Here, correct initialization means that before a mission starts, each agent adjusts its initial position and velocity to achieve the initial conditions of its input signal and input velocity. The advantages of the algorithm (10)-(12) are that the needs for relative velocity measurements and communication are removed. Thus, the algorithm relays on only local sensing that lowers the cost of the system by removing the communication devices and the system is robust to initialization errors. However, the trade-off of this algorithm is that it still requires absolute velocity measurements and is limited to bounded input signals with bounded velocities and accelerations.

Remark 4.4 Despite the dependence of the control gains' lower bounds on some global information in both algorithms in Sections 3 and 4, the gains themselves are constant and can be determined off-line before a mission starts. One conservative approach is to select large enough gains based on

the worst case scenario of the graphs (e.g., considering the finite number of all possible connected graphs and checking the eigenvalues of the Laplacian matrices in all cases). Also to obtain a better estimate of the lower bounds of the gains, it is possible to run an average consensus algorithm to estimate n and a max consensus algorithm to figure out the largest bounds of x_i^r , v_i^r and a_i^r , respectively.

5 CONCLUSIONS

Distributed average tracking for a group of double-integrator agents has been studied. First, a distributed controller combined with a distributed filter was proposed in the absence of velocity information and correct initialization to deal with input signals with bounded deviations among input signals, input velocities, and input acceleration. Here, only the relative position and filter output information were used in the control design besides each agent's own position and input signal information. Second, another algorithm was introduced to remove the requirement of communication under the assumption that the inputs, their velocities and their accelerations are bounded. The algorithm used absolute velocity information but not relative velocity information. In both cases, reduced requirement on velocity measurements was achieved and hence the cost for velocity measurements was reduced.

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