Band gap property analysis of periodic plate structures under general boundary conditions using spectral-dynamic stiffness method

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Abstract

Vibration band gap properties of periodic rectangular plate structures with general boundary conditions are studied using the spectral-dynamic stiffness method (S-DSM). Material and dimensional properties are assumed to vary periodically in the whole plate structure. In the S-DSM, the general solution governing each unit plate is firstly derived with the aid of spectral method and then the dynamic stiffness matrix is established by substituting the general solution into corresponding boundary equations. Finally, the global spectral dynamic matrix for the whole periodic plate structure is assembled in a similar way to the finite element method (FEM). The harmonic responses are calculated to illustrate the band gap properties of the periodic structures. By comparing the results obtained by the FEM, it can be verified that the S-DSM is of superior accuracy and convergence rate. A parametric study is also conducted to investigate the effects of the material properties, geometrical parameters and structural damping on the vibration band gap behaviors of the periodic structures.

Keywords: Spectral dynamic stiffness method; Vibration band gap; Periodic plate structure; General boundary conditions

1. Introduction

The propagation of elastic waves in periodic systems has received much attention over the years [1-8]. More recently, there has been an active area of research in a special type of inhomogeneous materials and geometries (the so-called phononic crystals) whose elastic coefficients or geometrical parameters vary periodically in space. Many remarkable mechanic properties of such structures have been discovered and investigated. Among them, the band gap property in frequency domain is one of the most distinctive properties which is caused by the elastic waves reflected back and forth between the interfaces of the adjacent cells when they propagate in the periodic structures. In the band gap frequency range, nearly no elastic wave can propagate through the periodic structure. The vibration band gap properties enable the periodic structure to be widely used in engineering fields, such as, frequency filters, noise controls, vibrationless environments for high-precision mechanical systems, etc. [4, 13, 14].

A variety of both theoretical and numerical studies have been conducted to investigate the wave filtering behavior in periodic structures. For example, Yan and Wang [9, 10, 12] developed a wavelet-based method (WBM) to study the elastic band gaps of phononic crystals. The eigen-equation for the phononic crystals was derived by expanding the displacement fields in the wavelet basis functions. Based on the transfer matrix method (TMM), Ruzzene and Marino [15, 16] studied the dynamic behaviors of the periodic rods with shape memory inserts which can act as the filters to weaken the elastic wave propagation along the rods. Yu et al. [17] extended the TMM to calculate the band gaps of flexural waves in the Euler-Bernoulli beam and studied its transverse vibration band gap behaviors. To investigate the flexural vibration band gap properties of the periodic beams, Xiang and Shi [18] derived the theoretical equations of beam by employing the Bloch-Floquet theorem and then solved the equations using the differential quadrature method.

Olhoff et al. [19] analyzed a repeated inner segment of the optimized beams using the waveguide finite element method (WFEM) to study the band-gap for travelling waves. The authors aimed to maximize the band gaps between two adjacent natural frequencies and to study the associated creation of periodicity of the optimized beam designs. Hussein et al. [20] designed the target frequency band structures characterizing longitudinal wave motion of one-dimensional periodic unit cell by using a multi-objective genetic algorithm. The design approach provided practical guides for forming bounded structures with advanced vibration, shock isolation and filtering characteristics. Considerable vibration studies on various of periodic cylindrical shell structures are also conducted in recent years [21-23]. Shen et al. [24] and Wen et al. [25] took the advantages of band gap properties of periodic composite pipes to provide a novel avenue for vibration control and noise reduction in complex piping systems. Sorokin and Ershova [26] employed the Floquet theory and boundary integral equations to study the steady-state free vibration of non-uniform elastic cylindrical shells with and without internal heavy fluid loading. Shen et al. [27, 28] illustrated the mechanism of wave propagation and vibration transmission in functionally graded material periodic cylindrical shell and studied its capability to control sound and vibration.

The structures of periodically placed plates are commonly encountered in various practical engineering. Much effort has been devoted to investigating the dynamic characteristic of such periodic plate structures in the past decades. In the early years, Sigalas and Economou [29] calculated the elastic waves propagation in plates with solid inclusions placed periodically in the host plate. Langley and Smith [30] used the statistical energy analysis (SEA) to study the effects of disorder in the stiffener spacing on the high frequency vibration transmission through a periodic plate system. In recent years, Wang et al. [31] developed an improved FEM to study the physical mechanisms for the formation of periodically stiffened plate structures with any number or

orientation of stiffeners. Wu et al. [32, 33] applied the spectral element method (SEM) to investigate the dynamic behavior of periodic plate structures with classic plate theory (CPT) and Mindlin plate theory (MPT). But both the improved FEM and SEM have their limitations. For the FEM, the number of element should be large enough to obtain credible results in high-frequency range. In addition, computational efficiency and memory space will be unacceptable if the element number becomes very large. Although the SEM is of excellent accuracy and efficiency, it is only suitable for limited boundary conditions for plate structures, i.e., at least two opposite simply supported boundaries. Recently, the dynamic stiffness method (DSM) has drawn more and more attention due to its high computational efficiency and accuracy compared with the FEM resulting from the exact shape functions and lower degrees of freedom. And it has been developed for a bunch of one-dimensional (1D) elements such as rods and beams [34-36] and two dimensional (2D) rectangular plates using various plate theories [37, 38]. However, for plate structures, the traditional DSM is only available for the cases where at least a pair of opposite boundaries are simply supported, i.e. Levy plates. More recently, Liu and Banerjee [39] developed a novel S-DSM by introducing the idea of general spectral method, which had removed the boundary constraints of the traditional DSM. This idea certainly opened up a new way to solve vibration problems of plate structures in an accurate and efficient manner. Afterwards, Banerjee and his co-workers extended and extended the S-DSM to study the free vibration of composite plate structures and plate assemblies with arbitrary non-uniform elastic supports mass attachments and elastic coupling constraints [40-42].

In the present work, the S-DSM is employed to calculate the forced vibration responses of periodic plate structures under general boundary conditions and investigate the band gap behaviors in frequency domain. The S-DSM is more efficient than the FEM and can cope with more complex

boundary conditions than the SEM. This paper is arranged as follow: in Section 2, the formulation of the dynamic stiffness matrix of the periodic plate structure is introduced briefly, after which the applications of external load and arbitrary prescribed boundary conditions are given. In Section 3, firstly, several numerical examples are given to validate the accuracy and convergence of the current results. Then, a parametric study is conducted to investigate the band gap behaviors of the periodic palate structures under the effects of material and dimensional properties. Finally, the conclusions of this paper are given in Section 4.

2. Mathematical formulation

The periodic plate structure shown in Fig. 1 is considered in this work. The system consists of N repetitions of alternating plates A and B . Plates A and B are made of different material properties or geometric dimensions. A set of parallel edges are under general boundary conditions and the other set of edges are free or coupled with the edges of the adjacent plates.

2.1 Governing differential equation

Fig.2 shows a rectangular Kirchhoff plate in its local coordinate system. The length, width and thickness of the plate element are $2a$, $2b$ and h , respectively. In this study, the plate is assumed to be homogeneous, isotropic, elastic and of uniform thickness. The thickness of the plate is much smaller than the other two dimensions, and the inertia of shear and rotation are supposed to be negligible. Then, based on the Kirchhoff thin plate theory, the differential equation can be described as [43]

$$D \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + \rho h \left(\frac{\partial^2 w}{\partial t^2} \right) = 0 \quad (1)$$

where $D = Eh^3 / [12(1-\nu^2)]$ is the bending stiffness, E is the Young's modulus, ν and ρ are the Poisson's ratio and density of the plate respectively.

By introducing the harmonic representation, the transverse displacement can be given as

$$w(x, y, t) = W(x, y)e^{i\omega t} \quad (2)$$

where ω is the angular frequency, $W(x, y)$ is the amplitude of the transverse displacement in frequency domain. By substituting Eq. (2) into Eq. (1), one can get

$$D \left(\frac{\partial^4 W}{\partial x^4} + 2 \frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4} \right) - \rho h \omega^2 W = 0 \quad (3)$$

Based on the thin plate assumptions, the rotations of the transverse normal, bending moments and transverse shear forces can be written as

$$\varphi_x = -\frac{\partial W}{\partial x}, \quad (4)$$

$$\varphi_y = -\frac{\partial W}{\partial y}, \quad (5)$$

$$M_x = -D \left(\frac{\partial^2 W}{\partial x^2} + \nu \frac{\partial^2 W}{\partial y^2} \right), \quad (6)$$

$$M_y = -D \left(\frac{\partial^2 W}{\partial y^2} + \nu \frac{\partial^2 W}{\partial x^2} \right), \quad (7)$$

$$V_x = -D \left(\frac{\partial^3 W}{\partial x^3} + (2 - \nu) \frac{\partial^2 W}{\partial x \partial y^2} \right), \quad (9)$$

$$V_y = -D \left(\frac{\partial^3 W}{\partial y^3} + (2 - \nu) \frac{\partial^2 W}{\partial y \partial x^2} \right), \quad (10)$$

2.2 General solution

In the S-DSM, the general solution of Eq. (3) can be sought according to the method of separation of variables [39]

$$W(x, y) = X(x)Y(y) = Ce^{qx+py} \quad (11)$$

Where C , q and p are arbitrary coefficient and wave numbers along x and y directions respectively.

By submitting Eq. (11) into Eq. (3), the general solution $W(x, y)$ of the governing differential

equation can be obtained based on Euler's formula. For further consideration of the symmetry/antisymmetry properties of the plate structure, the general expression of $W(x, y)$ can be split into four contributions: symmetric-symmetric (00), symmetric - antisymmetric (01), antisymmetric - symmetric (10), antisymmetric - antisymmetric (11) as

$$W(x, y) = \sum_{k, j \in \{0,1\}} W^{kj}(x, y) = W^{00} + W^{01} + W^{10} + W^{11} \quad (12)$$

where all the four parts of the solution take the unified concise form:

$$W^{kj}(x, y) = \sum_{m \in M} [A_{1km} \mathcal{H}_j(p_{1km} y) + A_{2km} \mathcal{H}_j(p_{2km} y)] \Xi_k(\alpha_{km} x) + \sum_{n \in N} [B_{1jn} \mathcal{H}_k(q_{1jn} x) + B_{2jn} \mathcal{H}_k(q_{2jn} x)] \Xi_j(\beta_{jn} y) \quad (13)$$

2.3 Dynamic stiffness matrix

2.3.1 Dynamic stiffness matrix for individual plate structure

The detailed formulation of the dynamic stiffness matrix for the individual plate structure has been detailedly deduced by Liu and Banerjee [39]. In this section, only the main strategy of the derivation procedure is introduced.

As given in Eq. (12), the general solution is split into four parts with different symmetry/antisymmetry properties, and likewise, the arbitrarily prescribed boundary conditions are also decomposed into four kj parts defined in Ω_1 (first quarter of Ω , see Fig.3 (a)) as shown in Fig. 3(b). Then, the boundary conditions are expressed using the modified Fourier series as

$$\begin{bmatrix} W_a^{kj} \\ W_b^{kj} \\ \phi_a^{kj} \\ \phi_b^{kj} \end{bmatrix} = \begin{bmatrix} \sum_{n \in N} W_{ajn}^{kj} \frac{\Xi_j(\beta_{jn} y)}{\sqrt{\zeta_{jn} b}} \\ \sum_{m \in M} W_{bkm}^{kj} \frac{\Xi_k(\alpha_{km} x)}{\sqrt{\zeta_{km} a}} \\ \sum_{n \in N} \phi_{ajn}^{kj} \frac{\Xi_j(\beta_{jn} y)}{\sqrt{\zeta_{jn} b}} \\ \sum_{m \in M} \phi_{bkm}^{kj} \frac{\Xi_k(\alpha_{km} x)}{\sqrt{\zeta_{km} a}} \end{bmatrix}, \quad \begin{bmatrix} V_a^{kj} \\ V_b^{kj} \\ M_a^{kj} \\ M_b^{kj} \end{bmatrix} = D \begin{bmatrix} \sum_{n \in N} V_{ajn}^{kj} \frac{\Xi_j(\beta_{jn} y)}{\sqrt{\zeta_{jn} b}} \\ \sum_{m \in M} V_{bkm}^{kj} \frac{\Xi_k(\alpha_{km} x)}{\sqrt{\zeta_{km} a}} \\ \sum_{n \in N} M_{ajn}^{kj} \frac{\Xi_j(\beta_{jn} y)}{\sqrt{\zeta_{jn} b}} \\ \sum_{m \in M} M_{bkm}^{kj} \frac{\Xi_k(\alpha_{km} x)}{\sqrt{\zeta_{km} a}} \end{bmatrix} \quad (14)$$

where $W_{ajn}^{kj}, W_{bkm}^{kj}, \phi_{ajn}^{kj}, \phi_{bkm}^{kj}, V_{ajn}^{kj}, V_{bkm}^{kj}, M_{ajn}^{kj}$ and M_{bkm}^{kj} are the Fourier coefficients.

On the basis of the relationships between the displacement and force given in Eq. (4-10), the unknown coefficients $\tilde{A}_{1km}$, $\tilde{A}_{2km}$, $\tilde{B}_{1jn}$ and $\tilde{B}_{2jn}$ in the general solution can be determined by substituting Eq. (13) into Eq. (14). Subsequently, an infinite sets of algebraic equations can be obtained by substituting the determined coefficients into Eq. (14), which can be summarized into matrix form as

$$\mathbf{f}^{kj} = \mathbf{K}^{kj} \mathbf{d}^{kj} \quad (15)$$

In Eq. (15), the Fourier coefficient vectors $\mathbf{f}^{kj}$ and $\mathbf{d}^{kj}$ are of the boundary conditions on B_a and B_a of the quarter plate as shown in Fig. 3(b). Similarly, the corresponding Fourier coefficient vectors of the force and displacement on the boundary lines of the entire plate can be given as

$$\mathbf{d} = [\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3, \mathbf{d}_4]^T, \quad \mathbf{f} = [\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4]^T \quad (16)$$

where

$$\mathbf{d}_i = [\mathbf{W}_i^0, \mathbf{W}_i^1, \Phi_i^0, \Phi_i^1]^T, \quad \mathbf{f}_i = [\mathbf{V}_i^0, \mathbf{V}_i^1, \mathbf{M}_i^0, \mathbf{M}_i^1]^T \quad (17)$$

Then, by introduction of the transfer matrix $\mathbf{T}$ [39], one can establish the relationships between $\mathbf{f}, \mathbf{d}$ and $\mathbf{f}^{kj}, \mathbf{d}^{kj}$ as

$$\mathbf{d} = \mathbf{T} [\mathbf{d}^{00}, \mathbf{d}^{01}, \mathbf{d}^{10}, \mathbf{d}^{11}]^T, \quad \mathbf{f} = \mathbf{T} [\mathbf{f}^{00}, \mathbf{f}^{01}, \mathbf{f}^{10}, \mathbf{f}^{11}]^T \quad (18)$$

Finally, the dynamic matrix $\mathbf{K}$ for the individual plate structure can be obtained by integrating Eq. (15-18), which is reconstructed in the following form

$$\mathbf{f} = \mathbf{K} \mathbf{d} \quad (19)$$

where

$$\mathbf{K} = \frac{1}{2} \mathbf{T} \begin{bmatrix} \mathbf{K}^{00} & \mathbf{O} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{K}^{01} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{K}^{10} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{K}^{11} \end{bmatrix} \mathbf{T}^T \quad (17)$$

2.3.2 Spectral dynamic stiffness equation of the whole periodic structure

In this section, the dynamic stiffness matrices for the individual plates are assembled into global matrix for the periodic plate structure in a manner similar with the FEM, except that the plates are connected along the boundary lines rather nodes. As a result, the equation of motion of the whole periodic plate structure can be obtained as

$$\mathbf{f}^g = \mathbf{K}^g \mathbf{d}^g \quad (21)$$

where $\mathbf{K}^g$ is the global spectral dynamic stiffness matrix, $\mathbf{d}^g$ and $\mathbf{f}^g$ are the displacement and force vectors in the global coordinate system.

2.4 Forced vibration excitation load

In this section, the application of the external excitation for the forced vibration is introduced. The force is assumed to be harmonically varying dynamic forces with the intensity of constant values.

Without loss of generality, the external normal force is located at boundary line 1 (as depicted in Fig. 4(a)) in this paper. Therefore, in the force vector $\mathbf{f}$, nonzero values only appear the vector $\mathbf{V}_1$ of $\mathbf{f}_1$ as follow

$$\mathbf{f} = [\mathbf{f}_1, \mathbf{0}, \mathbf{0}, \mathbf{0}]^T, \quad \mathbf{f}_1 = [\mathbf{V}_1^0, \mathbf{V}_1^1, \mathbf{0}, \mathbf{0}]^T \quad (22)$$

According to the modified Fourier series given in Eq. (14) and the relation of $\mathbf{f}$ and $\mathbf{f}^{kj}$ given in Eq. (16), the shear force vectors $\mathbf{V}_1^0$ and $\mathbf{V}_1^1$ can be expressed as

$$\begin{aligned} \mathbf{V}_1^0 &= \mathbf{V}_a^{00} + \mathbf{V}_a^{10} \\ \mathbf{V}_1^1 &= \mathbf{V}_a^{01} + \mathbf{V}_a^{11} \end{aligned} \quad (23)$$

where

$$\mathbf{V}_a^{kj} = [V_{aj0}, V_{aj1}, \dots, V_{ajn}, \dots]^T \quad (24)$$

In which, the Fourier coefficients of the boundary conditions V_{ajn} are determined as

$$V_{ajn} = \int_{-b}^b F(x, y) \frac{\Xi_j(\beta_{jn} y)}{\sqrt{\zeta_{jn} L}} dy \quad (25)$$

where $F(x, y)$ is the external load located along the boundary line.

2.5 Application of arbitrarily prescribed boundary conditions

For a general Kirchhoff's thin plate, the governing equation is a forth order partial derivative function, and only two boundary constraints are required at each point of the boundary. Three major kinds of common boundary conditions may be specified on the plate boundary conditions.

- Kinematic boundary conditions : prescribed displacement and rotation (clamped edge)

$$w(\mathbf{r}) - \bar{w}(\mathbf{r}) = 0, \quad \mathbf{r} \in \mathcal{B} \quad (26)$$

$$\phi(\mathbf{r}) - \bar{\phi}(\mathbf{r}) = 0, \quad \mathbf{r} \in \mathcal{B} \quad (27)$$

- Mechanical boundary conditions: prescribed bending moment and shear force (free edge)

$$m(\mathbf{r}) - \bar{m}(\mathbf{r}) = 0, \quad \mathbf{r} \in \mathcal{B} \quad (28)$$

$$v(\mathbf{r}) - \bar{v}(\mathbf{r}) = 0, \quad \mathbf{r} \in \mathcal{B} \quad (29)$$

- Mixed boundary conditions: prescribed displacement and bending moment (simply supported edge)

$$w(\mathbf{r}) - \bar{w}(\mathbf{r}) = 0, \quad \mathbf{r} \in \mathcal{B} \quad (30)$$

$$m(\mathbf{r}) - \bar{m}(\mathbf{r}) = 0, \quad \mathbf{r} \in \mathcal{B} \quad (31)$$

where $\mathcal{B}$ represents boundaries of the plate. $\bar{w}(\mathbf{r})$, $\bar{\phi}(\mathbf{r})$, $\bar{v}(\mathbf{r})$ and $\bar{m}(\mathbf{r})$ are the prescribed displacement, normal rotation, shear force and bending moment respectively, which are set as zero vectors in the following procedures for simplicity. The boundary conditions are applied to the S-DSM by eliminating the rows and columns corresponding to the potential zero elements appeared in the displacement vector $\mathbf{d}$.

3. Numerical results

In this part, the S-DSM is used to analyze the band gap properties appeared in the dynamic responses of the periodic plate structures. Firstly, an individual plate structure is adopted to validate the convergence and accuracy of the S-DSM. Then the vibration band gap properties in the periodic plate structures are mainly investigated by a number of numerical examples. To further study the accuracy and efficiency of the present method, the computational efficiency study is performed at last. For convenience, in the following, referring to the classical boundary conditions, F, S and C denote free, simply and clamped supported boundaries respectively.

3.1 Individual plate structure

In this section, the dynamic responses of the individual plate structure as shown in Fig. 4(a) are calculated. The structure and material parameters for the plate are taken as $a = 0.2$ m, $b = 0.2$ m, and the thickness $h = 0.01$ m, the Young's modulus $E = 200$ GPa, the Poisson ratio $\nu = 0.3$ and the mass density $\rho = 7800$ kg/m³. Two parallel boundaries along $y = \pm b$ are supposed to be free, clamped supported and simply supported, and the other two boundaries which $x = \pm a$ are free. A point force $F = F_0 e^{i\omega t}$ is applied on point $M_1 (0, -a)$, and F_0 is the amplitude of the force. In addition, the response point is located at $M_2 (0, a)$. A Matlab computer program is used to validate the S-DSM.

Since a limited number of terms in the general solution must be used for an actual calculation, the proposed solution in Eq. (13) should be truncated into finite terms. Thus, the convergence rate and computational efficiency should be regarded as the primary criterion of the present method. The convergence of solution for the individual plate structure with CFCF and SFSF boundary conditions are illustrated in Figs. 5 and 6, respectively. The number N in the legend represents the truncated number for the m and n in Eq. (15), which indicates the number of the terms of the modified Fourier

basis functions. It is clear that the results converge rapidly as the truncated number increases both for CFCF and SFSF boundary conditions. Even when the truncated number is selected as 3, the convergence can be guaranteed in frequency range from 0 to 3000 Hz. Furthermore, the calculation results are nearly identical when the truncated numbers M and N are fixed at 5 and 7. Figs. 7 and 8, respectively, show the comparisons of the dynamic responses of the individual plate structure under CC and SS boundary conditions. Good agreement of results obtained by the FEM and S-DSM models is obtained in lower frequency range. As the frequency increases, deviations appear between the results of the FEM with a coarse mesh (10×10) and the S-DSM. Nevertheless, the results of the finer meshed FEM (20×20) model show superior agreement with those of the S-DSM which can prove that the convergence of FEM towards to S-DSM. It also indicates that, if more accurate FEM results are desired in high frequency range, much finer mesh should be guaranteed firstly. In conclusion, the present method can deal with problems of higher frequency with small truncated number and keep excellent accuracy in contrast with the FEM.

3.2 Periodic plate structure

In this section, the vibration band gap properties of the periodic plate structures shown in Fig. 4(b) are investigated. Firstly, several examples are implemented to validate the present method by comparing with the FEM. Additionally, a systematic parameter study is conducted to research the effects of boundary conditions, materials and dimensional properties of the unit plate on the band gap behaviors. In the following procedures, the number of the unit plate is fixed equal to 15 and the periodic structure with epoxy as material A and steel as material B are investigated without special instructions. The physical parameters of the materials used in following examples are listed in Table 1.

3.2.1 Validation

In order to further validate the ability of the present method for solving the forced vibration problems of the periodic plate structures, several numerical examples are given in this subsection. A vertical point force $F = F_0 e^{i\omega t}$ is applied at $x = -a, y = 0$ as depicted in Fig. 4(b), where $a = b = 0.2$ m and $F_0 = 10$ N. The truncated number for the modified Fourier series is selected as $M = N = 5$. With regard to the periodic plate structure, only boundary lines parallel to the y axis are taken into consideration and those which are parallel to the x axis are considered as free or coupled with each other. For example, for the CS boundary condition, the first letter C indicates that the boundary line along $x=b$ are clamped supported, and the second letter S indicates the boundary line along $x=-b$ are simple supported.

Fig. 9 presents the comparison of the dynamic responses calculated by the S-DSM and FEM for the periodic plate structure with CC boundary condition. For comparison, two types of mesh (8×8 and 16×16 for a unit plate) are selected for the FEM. One can see that the present solutions are in good agreement with those obtained by the FEM. Meanwhile, it is noted that the solutions of 16×16 mesh FEM model are closer to the S-DSM results compared with those of the 8×8 mesh FEM model, particularly in high frequency scopes. That is, the FEM results converge to the S-DSM results.

In addition, Figs.10-13 give the dynamic responses of the periodic plates under CS, SS, CF and SF boundary conditions. The results of the S-DSM and FEM agree well with each other. Also, the band gaps can be observed obviously in the response curves for all the boundary conditions, especially in the lower frequency range. However, compared with response curves of the CF and SF boundary conditions, more and wider band gaps appear in the response curves of the CS and SS boundary conditions. In Fig. 12, only three obvious band gaps can be observed, and barely no band

gap appears in the frequency range from 700-1500 Hz. Although, many band gaps can be observed in the lower frequency range in Fig. 13, but the band gaps are very narrow and only one obvious band gap arises from 600-1500 Hz. Therefore, the F boundary condition is not conducive to the formulation of band gaps.

Figs. 14 and 15 show the transverse responses at each unit plate in the periodic plate structure for frequencies located at stop band and pass band respectively. In Fig. 14, the selected five frequencies are located in the first five stop bands respectively, i.e., $f = 206, 306, 476, 641$ and 866 Hz. On the whole, the vibration amplitudes decrease obviously as the number of the plate increases. The ratio of the vibration amplitudes of the first and the last plate is considerably large. Therefore, the vibration cannot propagate through the whole periodic plate when the harmonic frequency is located at the stop bands. On the contrary, Fig. 15 shows the responses for each unit plate for frequencies which are located at the pass bands, i.e., $f = 166, 246, 401, 561$ and 756 Hz. Despite the ups and downs, one can observe that the excitation under those frequencies at the left of the structure all propagate through the whole periodic structure.

3.2.2 Effect of material parameters

In this subsection, parametric investigations are implemented to study the effects of the material parameters on the band gap behaviors. For simplicity's sake, only CC and SS boundary conditions are taken into consideration. Figs. 16 and 17 depict the comparisons of the dynamic responses for periodic plates with different materials for the unit plate. In the figures, one can see that there is almost no band gap in the response curves for copper as material A and steel as material B. When the material B is aluminum, several small band gaps appear in the response curves. Furthermore, the response curves have clearer drops inside the band gaps when the materials A and B are epoxy and steel respectively. From the analysis, it can be concluded that the larger the

differences of the properties between materials A and B the more obvious band gaps appear in the response curves.

In practical engineering, the effects of structural damp are always non-negligible. In this work, structural damping is introduced in the present formulation by replacing Young's modulus E by a complex one $E = E (1 + i \eta)$, where η is the material loss factor. The effects of damping on the vibration responses of the periodic structure under SS boundary condition are illustrated in Fig. 18. It can be observed from Fig. 18 that with the introduction of the structural damping the locations of the band gaps remain unchanged. The comparison of the vibration responses of the damped and undamped structures indicates that the structural damping suppresses the amplitudes of the resonant peaks in the pass bands, especially in high frequency range. Meanwhile, the structure damping has little influence on the vibration amplitudes for non-resonant vibration.

3.2.3 Effect of geometrical parameters

In the following procedures, the band gap properties of periodic structures composed of unit plate with different geometric dimensions are studied. It is assumed that both the materials A and B of the unit plate in Fig. 4(b) are uniformly chosen as epoxy. The symbols h_1 and h_2 represent the thicknesses of plate A and B . In the calculations, h_1 increases from 0.008 m to 0.012 m with h_2 fixed at 0.005 m. The corresponding dynamic responses for the periodic structures with different thicknesses for unit plate are given in Figs. 19 and 20. As expected, more obvious band gaps appear as the h_1 becomes larger. However, it is a remarkable fact that, only when the ratio of h_1 and h_2 becomes very large, e.g. $h_1 / h_2 = 2.4$, obvious band gaps turn up in the dynamic response curves. This may be unacceptable in practical engineering. So, it's pretty obvious that the band gap properties can not only rely on the difference of geometrical dimensions between the plates. Therefore, the superposed effects of both the geometrical dimension and material property on the

band gap of the periodic plate structures are considered. For simplicity, only SS boundary condition is taken into consideration here. For comparison, Fig. 21(a) shows the dynamic responses of the periodic plate structure with different thicknesses and identical material for the unit plate and Fig. 21(b) shows the dynamic responses of the periodic plate structure with different thicknesses and different material properties. As one can see in Fig. 21 (a), no band gap appears in the dynamic response curves as h_1 increases. On the opposite, in Fig. 21(b), the band gaps become more and wider in the responses curves as h_1 changes with only a small drop. In conclusion, when the periodic structure is composed of plate with different materials it becomes much more sensitive to the dimensional variation of the unit plate. Small change can lead to significant effects on the band gap properties which has good reference to practical engineering.

Fig. 22 shows the frequency ranges of the first four stop bands with different ratio between the thickness and width of the unit plate. In the figures, the grey shaded parts represent the stop band of the vibration response and the corresponding colorless parts indicate the pass bands. When the ratio increases from 1/50 to 1/35, the bandwidth of the grey shaded part becomes larger but not particularly evident. On the other hand, the relative frequency ranges of the stop bands increase obviously as the ratio of $h/2a$ increases gradually. In conclusion, the thickness of the plate has not much influence on the bandwidth of the band gaps, but it affects the range of band gaps evidently.

Figs. 23 and 24, respectively, show the comparisons of dynamic responses for periodic structures with different unit plate numbers. In the calculation, the geometric dimensions of the individual plates are consistent and the materials of the structures are composed of epoxy and steel. It is clear that the frequency ranges of the band gaps have no change as the number of the plates increases from 5 to 10 to 15. But as the number of the unit plate increases, the stop bands become

deeper. As a result, the number of the unit cell in the periodic structure has no effect on the positions of the band gaps but has significant effects on the depths of the band gaps.

3.2.4 Computational efficiency

In order to further validate the accuracy and efficiency of the S-DSM, a computational efficiency study is performed. The dynamic responses of the periodic structure are calculated by the S-DSM with different truncated numbers and by the FEM with different mesh sizes, separately. Without loss of generality, only two response points are selected as illustration, i.e., one point at the middle and another on at the end of the structure. For comparison, the averaged error $\langle \varepsilon \rangle$ on forced responses of the two response points can be obtained as follow

$$\langle \varepsilon \rangle = \frac{1}{n_{rp}} \sum_{j=1}^{n_{rp}} \varepsilon_j \quad (56)$$

where

$$\varepsilon_j = \frac{\|w(\mathbf{r}) - w_{\text{ref}}(\mathbf{r})\|}{\|w_{\text{ref}}(\mathbf{r})\|} \quad (57)$$

where $n_{rp} = 2$. As references, the results of an S-DSM model with the truncated number $M = N = 9$ and an FEM model with mesh size equal to 0.005 are both taken into consideration. In practical calculation, the frequency range of the responses is from 200 to 300 Hz and the substep is selected as 0.5 Hz.

Fig. 25 plots the averaged relative prediction error of the normal displacement as a function of the calculation time. In Fig. 25(a), the results of an S-DSM model are adopted as the reference. Under the same averaged relative error, the S-DSM takes less time. Obviously, the S-DSM shows much higher computational efficiency. In Fig. 25(b), the results of the FEM model are selected as the reference and the S-DSM also performs better than the FEM. In addition, the S-DSM is

implemented as a research code in Matlab environment while the FEM is conducted in an optimized commercial program.

The S-DSM exhibits the excellent computational efficiency. To obtain the same computation error value, the computational time of the S-DSM is much less than that of the FEM.

4. Conclusion

In this paper, the band gap properties of the periodic plate structures with general boundary conditions are studied by using the S-DSM. In this paper, firstly, the computational accuracy and convergence rate of the S-DSM are validated. Then, numerous numerical examples are presented to investigate the band gap properties of the periodic plate structures with different boundary conditions. From the numerical results, the following conclusions can be drawn:

- (1) The S-DSM is employed to solve the forced vibration problems of the periodic plate structures and study the band gap properties appearing in the dynamic responses. Good agreement has been achieved by comparing the present results with those obtained by the FEM.
- (2) The material properties have great effects on the band gap behaviors of the periodic plate structures. Basically, the greater differences between the materials the more obvious band gap appears.
- (3) Only large scale differences in geometrical dimensions of the unit plate can lead to obvious band gap phenomenon. However, if the material properties of the unit plate are different, a small deviation of the geometrical dimensions can affect the width and location of the band gaps evidently.

(4) The structural damping only affects the magnitude of the nascent peaks in the pass band, especially in high frequency domain. Besides, the locations of the band gaps are hardly influenced.

This paper provides a detailed numerical investigation into the band gap properties of the periodic structures. Numerous numerical results provided in this paper can be used as reference for further researches and engineering design. In the future research, the S-DSM can be extended to investigate the vibration characteristic of periodic rib-stiffened plate structures, periodic sandwich panels with corrugated cores and so on.

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(b) Different material properties.

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(a) CC boundary condition.

(b) SS boundary condition.

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(a) S-DSM as reference

(b) FEM as reference

Fig. 25 Relative error as function of the calculation time.

Table:

Table 1. Physical parameters of the materials.

Materials	Young's modulus E	Density ρ
	(GPa)	(Kg/m ³)
Epoxy (Ep)	4.35×10 ⁹	1180
Steel (Fe)	200×10 ⁹	7800
Copper (Cu)	127×10 ⁹	8960
Aluminum (Al)	77.6×10 ⁹	2730